BUILDING THERMAL MODELS REDUCTION: IMPROVING EXISTING METHODS BY TAKING SPECTRAL INPUTS CHARACTERISTICS INTO ACCOUNT

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ABSTRACT

Modal-based model reduction techniques have been modified in order to improve their performance when applied to building thermal models. The basic idea is to take the spectral characteristics of meteorological inputs into account in such reduction techniques, which are very sensitive to the hypothesis concerning inputs nature. The resulting modified modal-based reduction methods have been implemented in Matlab and tested. From the performed analyses, it may be concluded that the introduced modifications significantly improve results.

INTRODUCTION

The user interfaces of building thermal simulation softwares now provide a rapid, complete representation of ever more detailed models. This trend has significantly increased the size of the differential systems to be solved. Model reduction has for this reason been an important line of research among thermal engineers since the beginning of the 1980s. The aim of reduction is to provide users with a simpler, but accurate system model, which will allow them to solve problems such as the simulation of the dynamic evolution of the system, or the calculation of control laws, etc., using a limited number of calculations.

Model reduction has been the subject of numerous investigations and a great number of bibliographic classifications have already been published. This paper addresses the reduction of linear, invariant models, in state form. Reduction techniques are widely used in the building thermal modelling field. The most usual ones are listed in table 1, where they are classified according to the applied reduction principle and the basis/transformation required.

Modal-based methods assume that inputs signals to the model are Heaviside step or Dirac impulse functions. On the contrary, no assuptions concerning inputs are required by the methods based on balanced transformations.

The pertinence of the different reduction methods above has been largely analyzed (c.f. Palomo et al., 1996). Generally, reduced models obtained by methods using balanced transformations are better than the ones obtained by modal-based methods. However, getting an internally balanced representation of a knowledge model is much more difficult than getting a modal one. The required computing time is higher, and numerical problems quickly appear when the size of the knowledge model increases.

	PRINCIPLES		
	Truncation	**Agregation**	**Minimization**
Modal Basis	Marshall (1966) Litz (1981) Chen (1989) Michailesco (1979)	Michailesco (1979) Oulefki (1993)	
Balanced Transformation	Moore (1981)		Glover (1984)

Table 1. Usually used methods for thermal models reduction.

The aim of this investigation is to improve the performance of modal-based reduction methods when applied to building thermal models. That is, to approximate the quality of reduced models obtained by modal-based techniques to the one of the models obtained by reduction methods based in balanced transformations. The basic idea is to take the spectral characteristics of the meteorological inputs (solar radiation and outside air temperature) into account in modal-based reduction methods, which are very sensitive to the hypothesis concerning inputs.

We focussed our attention in modal methods based on agregation principles, which lead to better results than the ones based on truncation. The method of Michaïlesco (1979) and the amalgam method (Oulefki, 1993) are then examined herein.

LINEAR AND INVARIANT BUILDING THERMAL MODELS

We are looking at thermal systems which are invariant, linear, and reciprocal. The energy conservation equation after spacial discretisation can be written as:

$$C\,\dot{T}(t) = A\,T(t) + E\,U(t)$$
$$Y(t) = J\,T(t) + G\,U(t) \qquad (1)$$

where T(t) is the vector [*n*] of the nodal temperatures, *C* is the matrix [*n*, *n*] of the heat capacities at the discretisation nodes, *A* is the matrix [*n*, *n*] of the heat conductances between the internal nodes, *E* is the matrix [*n*, *p*] of the thermal coupling between the system and its environment, and U(t) is the vector [*p*] of the applied sollicitattions. *J* and *G* are two matrices [*q*, *n*] and [*q*, *p*] respectively. In the particular case where Y(t) = T(t), then $J = I_n$ (Identity matrix of order n) and *G* is the zero matrix.

Splitting $T(t) = T^g(t)+T^d(t)$ in a quasi-steady term, $T^g(t) = -A^{-1}E$, and in a dynamical one, $T^d(t)$, is a common practice in thermal analysis. The equations (1) can be then written as:

$$C\,\dot{T}^d(t) = A\,T^d(t) + CA^{-1}E\,\dot{U}(t)$$
$$Y(t) = J\,T^d(t) + \left[G - JA^{-1}E\right]U(t) \qquad (2)$$

Let us consider vector X(t) issued from the transformation of $T^d(t)$ by a regular matrix P:

$$T^d(t) = P\,X(t)\,; \qquad \det P \neq 0 \qquad (3)$$

Equation (2) becomes then:

$$\dot{X}(t) = F\,X(t) + B\,\dot{U}(t)$$
$$Y(t) = H\,X(t) + S\,U(t) \qquad (4)$$

with $F = P^{-1}(C^{-1}A)\,P$, $B = P^{-1}A^{-1}E$, $H = J\,P$ and $S = G - JA^{-1}E$. Systems (2) and (4) are equivalent, as long as basis transformation matrix P is regular. The most popular basis/transformation for thermal models, and the one that we are using here, is the modal one. It is defined by the following eigenvalue problem (cf. Lefebvre, 1987; El Khoury, 1989):

$$C^{-1}A = P\,F\,P^{-1} \qquad (4)$$

where F is a negative definite diagonal matrix made of the eigenvalues of the matrix $C^{-1}A$ ($\lambda_1, \lambda_2, ..., \lambda_n$), and P is the matrix [*n*, *n*] of the corresponding eigenvectors placed column wise.

STATISTICAL MODELLING OF THE METEOROLOGICAL INPUTS OF BUILDINGS

The principle of this modelling is to analytically express the outside air temperature and solar flux signals, in a simplified fashion. The criterion we used to determine whether the approximate signal adequately reflects the real signal is the power spectral density. Indeed, it can be shown (Palomo et al., 1997), that the type of inputs is involved through the power spectral density in the calculation of the reduced model by modal-based techniques.

The observation of the spectral densities of real signals shows the existence of strong periodicities in the temperature series, T(t), and solar radiation series, $\phi(t)$, and suggest the breakdown of these signals into a deterministic part, $T_d(t)$ and $\phi_d(t)$, supposed to describe the observed periodic behaviour, and a residual part ε (t) which can be considered to be stochastic. This gives

$$u(t) = u_d(t) + \varepsilon(t); \quad u = T, \phi \qquad (5)$$

This practice is commonly used in the modelling of daily and hourly series of temperature and solar radiation, which have formed the subject of many investigations. The way in which the deterministic and residual components are defined varies widely, however. In the work of (Brinkwoth, 1987; Goh, 1987; Paasen, 1987; Sihhuan, 1989; Scartezzini, 1989), for example, the residual part is considered as being the fluctuation of the signal relative to its time-dependent mean value. On another hand, different procedures for series normalization are used by (Boileau, 1983; Amanto, 1986; Balouktsis, 1989; Palomo, 1993; Palomo, 1995) to define the residues. In the case of solar radiation series, special indices were used. (Mustacchi, 1979; Graham, 1987; Graham, 1988; Aguiar, 1988; Aguiar, 1992; Gordon, 1989) thus use the clarity index to define the residues, and (Guinea, 1988, Palomo, 1989; Shihuan, 1989; Scartezzini, 1989) the cloudy state index.

The model adopted here to represent the outside air temperature is:

$$\begin{cases} T(t) = T_d(t) + \varepsilon(t) \\ T_d(t) = a_0 + 2 * \sum_{m=1}^{3} a_m \cos(w_m t) + b_m \sin(w_m t) \\ \ddot{\varepsilon}(t) + a_1\,\dot{\varepsilon}(t) + a_2\,\varepsilon(t) = z(t) \end{cases} \qquad (6)$$

with as harmonics (ω_1, ω_2 and ω_3), those corresponding to the frequencies $24h^{-1}$, $12h^{-1}$ and $6h^{-1}$. a_m and b_m are the corresponding Fourier coefficients. The stochastic residual part is represented by a 2nd order auto-regressive model, with $\zeta(t)$ being a white noise.

Figure 1 shows the power spectral density of temperature data for the month of January in Carpentras, as well as the spectral density estimated from the above model. It can be seen that the proposed model reproduces the spectral characteristics of the data fairly well. The same quality of results was obtained for all the temperature series analyzed (Nice and Trappes).

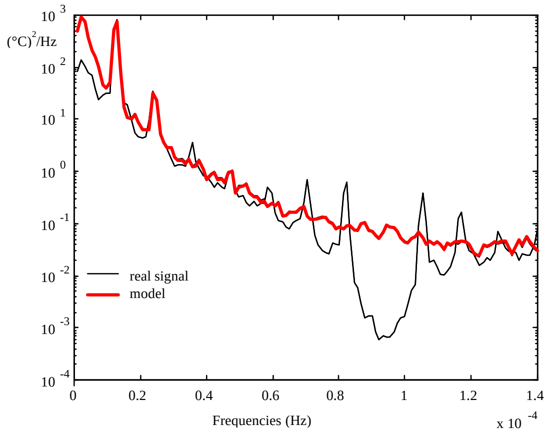

Figure 1. Power spectral density of series of hourly temperature: estimated from the data and calculated from the model.

The model adopted to represent the solar flux is:

$$\begin{cases} \phi(t) = f_d(t) + e(t) \\ f_d(t) = a_0 + 2 * \sum_{m=1}^{4} a_m \cos(w_m t) + b_m \sin(w_m t) \\ \dot{e}(t) + a_1 e(t) = z(t) \end{cases} \quad (7)$$

with as harmonics (ω_1, ω_2, ω_2 and ω_4), those corresponding to the frequencies $24h^{-1}$, $12h^{-1}$, $8h^{-1}$ and $6h^{-1}$. a_m and b_m are the corresponding Fourier coefficients. The stochastic residual part is represented by a 1st order auto-regressive model.

Figure 2 shows the power spectral density of solar flux data for the month of January in Carpentras, as well as the spectral density estimated from the above model. It can be seen that the proposed model reproduces the spectral characteristics of the data fairly well. The same quality of results was obtained for all the solar flux series analyzed (Nice and Trappes).

MODIFICATION OF MODEL REDUCTION TECHNIQUES

Given the following state system:

$$\begin{cases} \dot{X}(t) = F\,X(t) + B\,\dot{U}(t), \quad X \in \Re^n \\ Y(t) = H\,X(t) + S\,U(t) \end{cases} \quad (8)$$

the objective of the reduction is then to obtain a reduced model:

$$\begin{cases} \dot{\tilde{X}}(t) = \tilde{F}\,\tilde{X}(t) + \tilde{B}\,\dot{U}(t), \quad \tilde{X} \in \Re^r, \quad r << n \\ \tilde{Y}(t) = \tilde{H}\,\tilde{X} + \tilde{S}\,U(t) \end{cases} \quad (9)$$

reproducing correctly the the dynamic behaviour of the previous one.

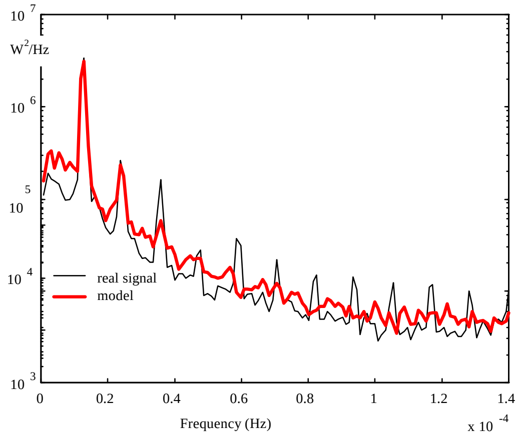

Figure 2. Power spectral density of hourly series of solar flux: estimated from the data and calculated from the model.

<u>Principle of modal agregation methods</u>

Let's take the example of the Michaïlesco technique to illustrate the principals of the modal reduction methods based on the agregation principle. The steps in this method are as follows:

- Transform the initial model into a modal state model.
- Calculate the grammian of the state variables X(t) of the modal model. Assuming inputs to be step functions, this is made by solving the following Lyapunov equation (c.f. Palomo, 1997):

  $$F\,W + W\,F^T + BB^T = 0$$
- Estimate the contribution of each one of the modes to the total energy of the system. It is given by the diagonal elements of the matrix $M = H^T H \otimes W$. The dominant modes are those showing a greater contribution to the system energy. Although the amalgam method uses a more elaborate criterion to decide about modes dominance, it is also based on W analysis.
- Obtain the matrices $\tilde{F}$ and $\tilde{B}$ by truncation: $\tilde{F} = L\,F$; $\tilde{B} = L\,B$. Matrix L, formed by zero and unity values, allows to select the rows and colums of F and B associated to the dominant modes.

- Obtain the matrix $\tilde{H}$ by minimization, with or without constraints, of a quadratic function of the reduction error. In the Michailesco method, the function to be minimized is the Frobenius norm of the reduction error

$$M = \int_t e^T(t)\, e(t)\, dt\ ; \quad e(t) = Y(t) - \tilde{Y}(t)$$

and the matrix $\tilde{H}$ is given by

$$\frac{\P M}{\P \tilde{H}} = 0 \;\Rightarrow\; \tilde{H} = (H\, W\, L^T)\,(L\, W\, L^T)^{-1}$$

Notice that the hypothesis concerning the type of control U(t) (impulse or step) is only involved in the calculation of the states covariance matrix W.

Modified techniques

The only change to be made in modal reduction methods to take into account the spectral characteristics of the meteorological inputs, involves then the calculation of the covariance matrix of the initial modal states.

Let

$$U(t) = U_{fourier}(t)+U_{markov}(t) \qquad (10)$$

be the inputs vector to the building model. $U_{fourier}(t)$ represents the deterministic part of the input signals, and $U_{markov}(t)$ the stochastic one. The components of $U_{fourier}(t)$ are described by means of Fourier series, and those of $U_{markov}(t)$ are supposed to be markov processes (e.g. statistical models 6 and 7).

It has been demostrated in (Palomo, 1997) that the covariance matix of the modal states X(t) is then given by

$$W = W_{fourier} + W_{markov} \qquad (11)$$

where $W_{fourier}$ represents the covariance matrix of the modal states when $U(t)=U_{fourier}(t)$, and W_{markov} the covariance matrix of the modal states when $U(t)=U_{markov}(t)$.

For inputs described by Fourier series

$$U_{fourier}(t) = \sum_m U_m\, e^{j\omega_m t} \qquad (12)$$

the covariance matrix of the states X(t) is given by (Palomo, 1997):

$$W_{fourier} = \sum_m \left[\Omega_m\, B\, U_m\right] \left[\Omega_m\, B\, U_m\right]^{*T} \qquad (13)$$

with

$$\Omega_m = \mathrm{diag}\left[\beta_{1m}\ \beta_{2m} \cdots \beta_{pm}\right];\quad \beta_{km} = \left[\frac{1 - j\dfrac{\lambda_k}{\omega_m}}{1+(\dfrac{\lambda_k}{\omega_m})^2}\right]$$

The components of the vector $U_m = \{u_{im}\}$ are related to the Fourier coefficients of the corresponding time series by $u_{im} = a_{im} - j\, b_{im}$.

A general continuos *m*-order markovien process is represented by

$$u^{(m)}(t) + \sum_{k=1}^{m} a_{m-k}\, u^{(m-k)}(t) = z(t) \qquad (14)$$

where $u^{(m)} = d^m u/dt^m$. Using a canonical internal representation, equation (14) becomes:

$$\dot{Z}(t) = \boldsymbol{\alpha}\, Z(t) + \mathbf{1}\, z(t) \qquad (15)$$

with $z_1 = u$, $z_2 = u^{(1)}$, ..., $z_m = u^{(m-1)}$. Matrices $\boldsymbol{\alpha}$ [*m*, *m*] and $\mathbf{1}$ [*m*, *1*] are given by:

$$\boldsymbol{\alpha} = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & & . & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ -a_0 & -a_1 & \cdots & \cdots & -a_{m-2} & -a_{m-1} \end{bmatrix};\ \mathbf{1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$$

Let $U_{markov}(t)$ be composed by *p* markovian processes, whose time evolution is governed by equations of type (15). A new system of ordinary differential equation can be then formed combining the the dynamical equations of the initial modal model (eq. 4) with the canonical internal representation of the inputs (eq. 15):

$$\dot{\eta}(t) = \Omega\, \eta(t) + \Xi\, \zeta(t) \qquad (16)$$

where $\eta(t) = \left[X \quad Z_1 \quad \cdots \quad Z_p\right]^T$. The blocks matrices Ω and Ξ are:

$$\Omega = \begin{bmatrix} F & F_1 & \cdots & \cdots & F_p \\ \mathbf{0} & \boldsymbol{\alpha}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{\alpha}_p \end{bmatrix};\ \Xi = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{1}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1}_p \end{bmatrix}$$

with dimensions $[n+\sum_{i=1}^{p} m_i ,\ n+\sum_{i=1}^{p} m_i]$ and $[n+\sum_{i=1}^{p} m_i ,\ p]$ respectively. n is the ordre of the initial model, p the number of inputs to the model, and m_i the order of the i-th markovien process. The matrix F_i $[n, m_i]$ is given by:

$$F_i = [\mathbf{0}_{n,1}\ B_i\ \mathbf{0}_{n,m_i-2}]$$

where $\mathbf{0}_{k,s}$ represents a $[k,s]$ zero matrix, and B_i the i-th column of the B matrix in (4).

As $\zeta(t)$ in (16) is a vector of white noise signals, the covariance matrix of the new vector of states $\eta(t)$ is the solution of the Lyapunov equation

$$\Omega\, W_{\eta\eta} + W_{\eta\eta}\, \Omega^T + \Xi\, N\, \Xi^T = 0 \qquad (17)$$

The covariance matrix of the initial modal model states W_{markov} $[n, n]$, is then given by (Palomo, 1997)

$$W_{hh} = \begin{bmatrix} W_{xx} & W_{xu} \\ W_{ux} & W_{uu} \end{bmatrix} ; \quad W_{markov} = W_{xx} \qquad (18)$$

RESULTS

A monozone building from the library in (Lefebvre, 1987) has been choosen for illustrating the contribution made by the proposed modifications of the modal reduction methods. Other examples are covered in (Dautin, 1997).

A modal model of this building was built using the *m2m* software (Lefebvre, 1996). It is a 33rd order model, with two inputs (solar flux and outside air temperature) and one output (the inside air temperature). The longest time constant of the model is 79 h 40 min and the shortest one 1.3 sec. The static gain values are 1 for the temperature input and 0.0425 for the solar flux input.

In table 2 are listed the calculated reduced models (3rd and 5th order) by Michaïlesco and amalgam methods. For each method, the three following hypothesis concerning inputs are considered:

- Inputs are step functions. Reduction is carried out by conventional methods.
- Inputs time evolution is governed by models (6) and (7). The reduction procedure proposed in this article is applied.
- Inputs are actual series of hourly data. In this case, the detailed model to which real inputs are applied is simulated. The state vector is thereby obtained and the state covariance matrix can be numerically obtained. This matrix is then incorporated into the reduction techniques.

	Amalgam	**Michaïlesco**
Step Input	Ama.Step.3 Ama.Step.5	Mic.Step.3 Mic.Step.5
Approximate Real Inputs (models 3&4)	Ama.Model.3 Ama.Model.5	Mic.Model.3 Mic.Model.5
Real Inputs (hourly data)	Ama.Real.3 Ama.Real.5	Mic.Real.3 Mic.Real.5

Tab. 2. Calculated reduced models.

The reduced model above were evaluated by comparing simulations from the initial model, Y(t), against those of the reduced models $\tilde{Y}(t)$. The applied control law is shown in figures 3 and 4. It involves meteorological data of the month of January in Carpentras.

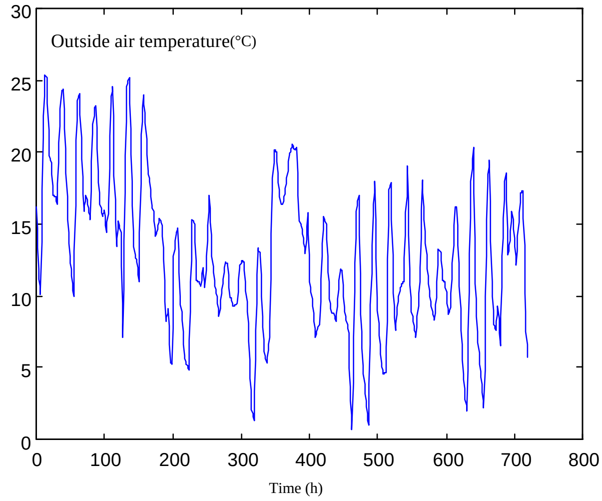

Figure 3. Outdoor air temperature. Month of January. Carpentras.

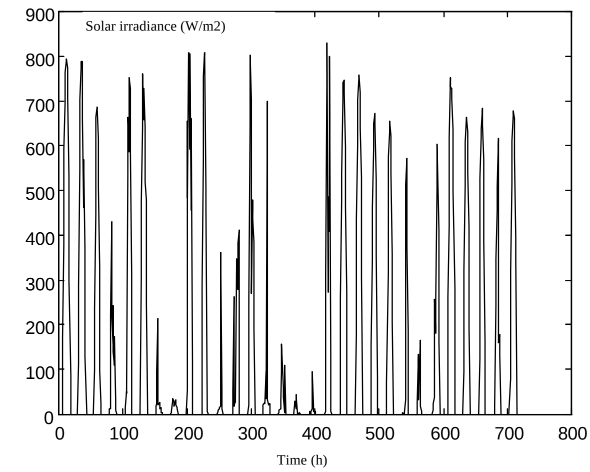

Figure 4. Hourly solar radiation data. Month of January. Carpentras.

The simulation error due to the reduction ($e(t) = Y(t) - \tilde{Y}(t)$), is shown in figures 5 and 6 (reduction by the Michaïlesco aggregation method of orders 3 and 5 respectively) and 7 and 8 (reduction by the Oulefki amalgam method of orders 3 and 5). It can be seen that if meteorological type inputs are assumed in all cases, there is a significant decrease in the error compared with the case in which step type inputs are assumed.

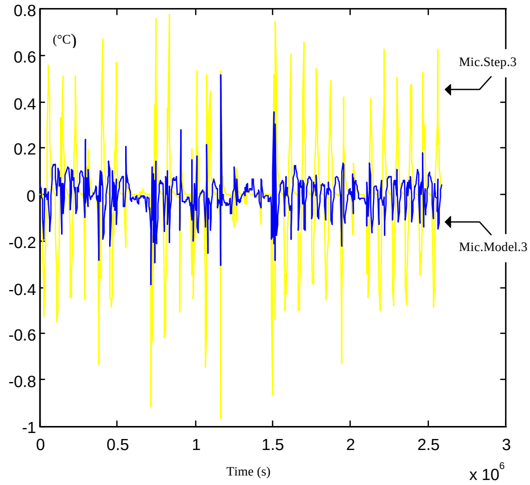

Figure. 5: 3rd order Michaïlesco reduced models.

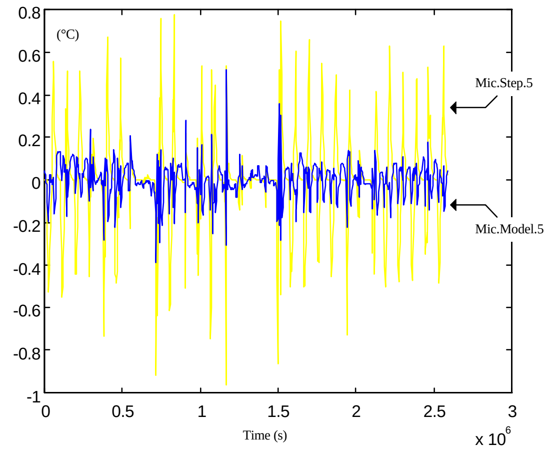

Figure 6. 5th order Michaïlesco reduced models.

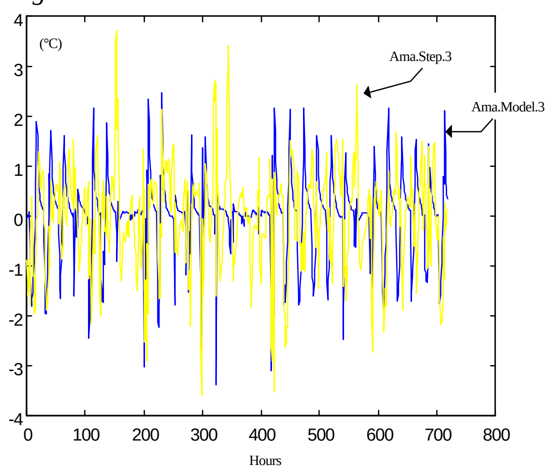

Figure. 7: 3rd order Amalgam reduced models.

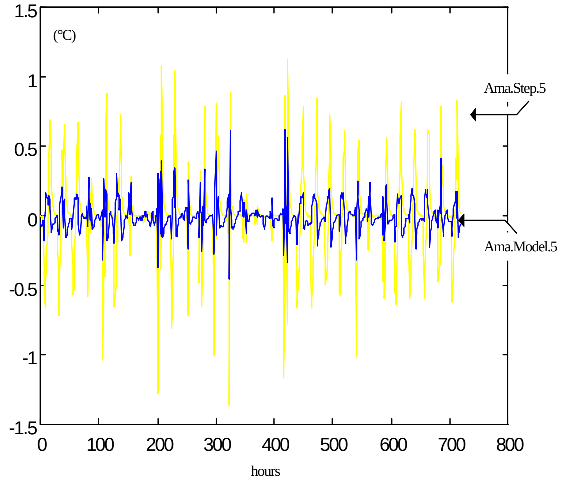

Figure. 8: 5th order Amalgam reduced models.

The maximum and standard deviation of errors are shown in the following table. The mean is not shown as it is very low in all cases.

Model	**Standard deviation**	**Maximum**
Mic.Step.3	0.4	1.6
Mic.Model.3	0.22	1.1
Mic.Real.3	0.22	1.1
Mic.Step.5	0.22	0.77
Mic.Model.5	0.08	0.5
Mic.Real.5	0.08	0.5
Ama.Step.3	0.8	2.47
Ama.Model.3	0.97	3.7
Ama.Real.3	0.97	3.7
Ama.Step.5	0.28	1.1
Ama.Model.5	0.1	0.6
Ama.Real.5	0.1	0.6
Moore.3	0.1	0.35
Moore.5	0.009	0.033

Tab. 3 Methods comparison

It can be seen that:

- The results are significantly improved when meteorological inputs in the Fourier+Markov form are taken into account. Only one case does not exhibit this property: the 3rd order amalgam. With the 5th order, however, it can be seen that the trend is reversed.
- The results with the approximate real inputs (models 6 and 7) and the real inputs obtained from data files are very close. The latter represent the best result that can be obtained by modifying the way inputs are taken into account. The difference between the reduced and detailed models in the case of the 3rd order Michaïlesco method is shown hereafter:

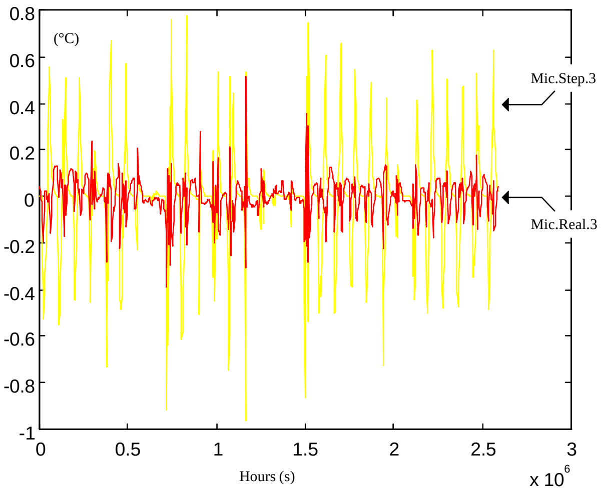

Fig.9.3rd order Michailesco reduced models (step focntions and actual data as inputs).

- When the order of the method is increased, the error e(t) decreases for all the techniques. The improvement in techniques through the taking into account of meteorological inputs, is also more significant.
- Although the method of Moore provides better results, the modification of modal method provides results which are not far away.

CONCLUSION

This investigation allowed us to characterize the meteorological inputs of building thermal response systems: outside air temperature and solar flux. We have shown that a very simple statistical model reproduces the spectral characteristics of the data fairly well.

We also showed how best to take into account these inputs models in modal-based model reduction techniques. Tests of these technique modifications were tested on two methods: The Michaïlesco aggregation and the Oulefki modal amalgam. It may be concluded from the performed analyses that the proposed modification significantly improves results. The error variation amplitude is significantly decreased, as well as the error maximum value, which is of great importance for control/monitoring studies.

The modified reduction methods provide results which are close to those of the Moore method. Although this method provides better results, the modification of modal-based techniques provides results which are not far away.

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