NEW EDUCATIONAL SOFTWARE FOR TEACHING THE SUNPATH DIAGRAM AND SHADING MASK PROTRACTOR.

John K.W. Oh and Jeff S. Haberl, Ph.D., P.E.
Energy Systems Laboratory, Department of Architecture,
Texas A&M University, College Station, Texas

ABSTRACT

The well-known versions of the sunpath diagrams that appear in the AIA's Architectural Graphics Standards are based on the equidistant sky dome projections and use a shading mask protractor developed by Olgyay and Olgyay at Princeton University in the 1950s. A designer using the AIA's Graphics Standards book, or other printed versions of the sunpath diagram, must select the nearest latitude, make photocopies of the appropriate sunpath diagram and shading mask protractor, and then overlay the shading mask protractor upon the diagram in the proper orientation. The outline of the shading device is then transcribed upon the shading mask, aligned at the proper orientation for the facade in which the window is being analyzed, and placed on top of the sunpath diagram to determine if a point centered at the base of the window is exposed to direct sunlight.

Teaching this process to architects and engineers is tedious and error-prone since the students must calculate several angles and then mentally translate their cartesian coordinates onto a sperical coordinate system to determine whether or not their shading device is going to have the intended effect. As a result of this, the sunpath diagram and shading mask protractor are not widely used because many designers either do not understand how to use the tools or do not budget the time to analyze a shading device properly with these tools.

This paper describes the new MS-Windows-based educational software package (Oh and Haberl 1996) that has been developed to fast-track the learning of the sunpath diagram and shading mask protractor which is based on previously published equations for plotting the sunpath diagram and shading mask protractor (McWatters and Haberl 1994a, 1994b, 1995). A review of the manual process is also provided to compare the computerized tool to the traditional design method.

INTRODUCTION

The sunpath diagram and shading mask protractor are well known graphic formats that have traditionally been used by architects and engineers to analyze whether or not a solar shading device will block direct sunlight on an exterior window. The sunpath diagram is a two-dimensional graphical representation of the movement of the sun across the sky's hemispherical vault for a given latitude. In the sunpath diagram the three-dimensional sky dome is flattened onto a two-dimensional circular chart where the sun's path becomes a series of elliptical curves that varies for different latitudes and time of the year.

When a shading mask protractor template is super-imposed upon the sunpath diagram, and oriented with respect to the proper off-south orientation to represent a window in a particular building facade, the combined diagrams indicate exactly those times of the year, and time of day when direct sunlight does or does not strike a point centered at the lower edge of the window. The diagrams also allow for the impact of shading devices, such as overhanging protrusions from the building (i.e., fins or eyebrows), to be accurately drawn with the shading mask protractor and superimposed on the sunpath diagram. The shading from adjacent buildings and nearby foliage can also be evaluated.

The origin of the sunpath diagram does not lie in the field of architecture. According to Olgyay and Olgyay (1957) it was first used by astronomers more than 400 years ago who published their early format in the 1531 "Rudimenta Mathematica" by Basel. The primary purpose of these early sun graphs was their use as a concise graph that indicated the sunrise and sunset times throughout the year at a given latitude. In general, research concerning the plotting and analysis of shading devices peaked during the 1930 to 1960 time period.

Various devices for physically measuring the effect of different shading devices on a window were developed including the heliodon, and thermoheliodon at Princeton University, and sun machine at the University of Kansas (Olgyay and Olgyay 1957). Interest picked up again during the later 1970s and early 1980s at the height of the solar years. During the 1980s interest in shading analysis has waned as the government funding for solar research plummeted. Alternative methods for plotting the sunpath diagram include the cylindrical plots by Mazria (1979) and orthographic representations of the sky dome. Additional reading on sunpath diagrams can be found in the solar text by Duffie and Beckman (1991), the AIA graphic standards (Ramsey and Sleeper 1994), or in the appendix to the environmental systems book by Stein and Reynolds (1992).

The well-photocopied versions of sunpath diagrams and shading mask protractors that appear in the AIA's Architectural Graphic Standards (Ramsey and Sleeper 1994) are based on the equidistant projections and the shading mask protractor which the Olgyays developed [1]. A designer using the AIA's Graphics Standards book must select the nearest latitude, make photocopies of the appropriate sunpath diagram and shading mask protractor, overlay the shading mask protractor upon the diagram and rotate the shading mask protractor until the proper off-south orientation is obtained. The outline of the shading device is then drawn using the shading mask protractor and aligned at the proper off-south orientation for the facade in which the window is being analyzed. Careful attention must be paid to select the correct construction lines on the shading mask protractor in order to accurately represent a particular shading device.

THE TRADITIONAL METHOD OF USING THE SUNPATH DIAGRAM AND SHADING MASK PROTRACTOR.

Figure 1 shows the sunpath diagram for a northern latitude of 30.6 degrees and indicates a target shading time of 10:00 a.m. to 4:00 p.m. during the cooling season (i.e., March through September for central Texas). In the traditional analysis the next step is to transcribe the sunpath onto the shading mask protractor so that it is aligned to represent the off-south azimuth of a particular window, in this case one that is 30 east of south as shown in Figure 2. In Figure 2 the outline of a window is also shown for a window that is 4 ft wide by 3 ft in height.

The next step in manual process is to assume a shading type and then begin the mental transformations that are required to change the x-y-z cartesian design coordinates into flattened spherical coordinates that can be projected using the shading mask protractor.

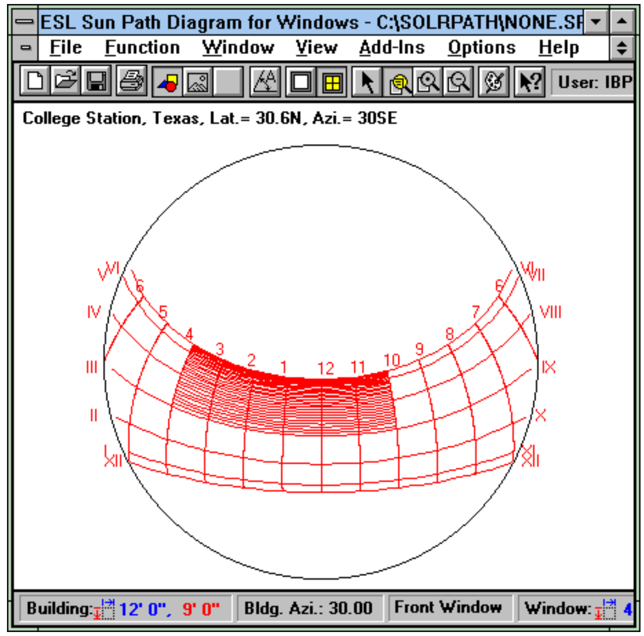

FIGURE 1: Sunpath diagram for a northern latitude of 30.6 degrees with a target shading time of 10:00 a.m. to 4:00 p.m. indicated from March through September.

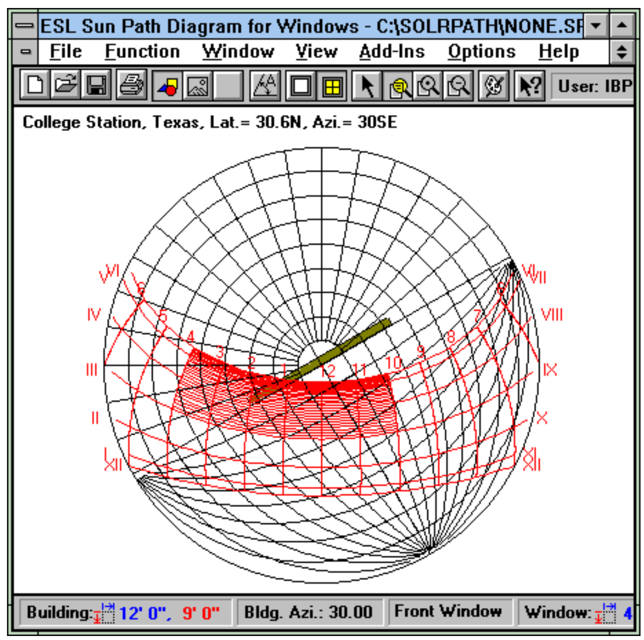

FIGURE 2: Sunpath diagram and rotated shading mask protractor. In this view the shading mask protractor has been added to the sunpath previously shown in Figure 1 and rotated 30 degrees east of south. The window frame for a window that is 4 ft in width by 3 ft in height is also shown.

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[1] There are several techniques for plotting the sun's path upon a flattened 2-D projection including orthographic projection, stereographic projection, and equidistant projection. The actual construction lines for the shading mask protractor have been physically verified using photographs taken with a "fisheye" lens, or globoscope which can be superimposed directly upon the sunpath diagram. Illustrations of this device are shown in Olgyay's book on pp. 46-47. The term "equidistant" refers to the equidistant spacing of the altitude lines (i.e., concentric circles) on the sunpath diagram.

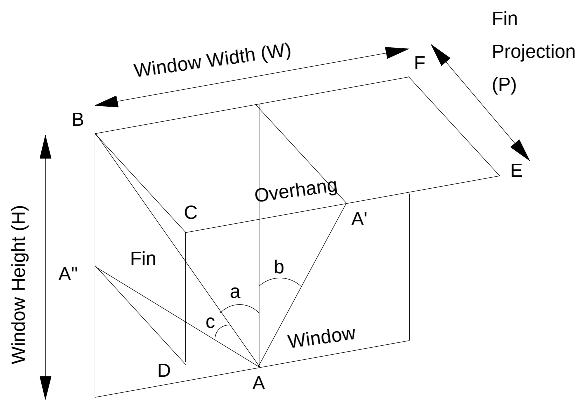

Figure 3: Example solar shading device that consists of an overhang and a right vertical fin.

If the shape is assumed that is shown in Figure 3 the next step is to determine the angle “a” that the width of the window would project upon the sunpath diagram. This is shown in Figure 2 as the solid bar in the center of the shading mask protractor that extends 33.7 degrees on either side of the center of the diagram (i.e., straight overhead). This can be calculated using the tangent relationship of one-half of the window width (W/2) and the height of the window (H).

$$\tan^{-1} a = \frac{W/2}{H} = \frac{2}{3} = 33.7^{o} \qquad (1)$$

The next step is to determine the angle “b” that is needed for the overhang to cover the sun’s path directly overhead and in front of the window (i.e., straight out from the window directly overhead). This is indicated on Figure 4 for an overhang that projects 2 ft. directly out from the top of the window. To determine this manually, one would need to assume a projection, determine the resultant angle, and transcribe the angle onto the shading mask protractor to see if it covered the intended portion of the sun’s path. This can be done using the previous relationship where the projection (P) and height (H) are evaluated as such:

$$\tan^{-1} b = \frac{P}{H} = \frac{2}{3} = 33.7^{o} \qquad (2)$$

Finally, as can be seen in Figure 4, there is a small portion of the intended shading that is not covered by the overhang alone. This can be accomplished in several ways, including extending the horizontal shade to the right, or by using a vertical fin that projects 2 ft. out from

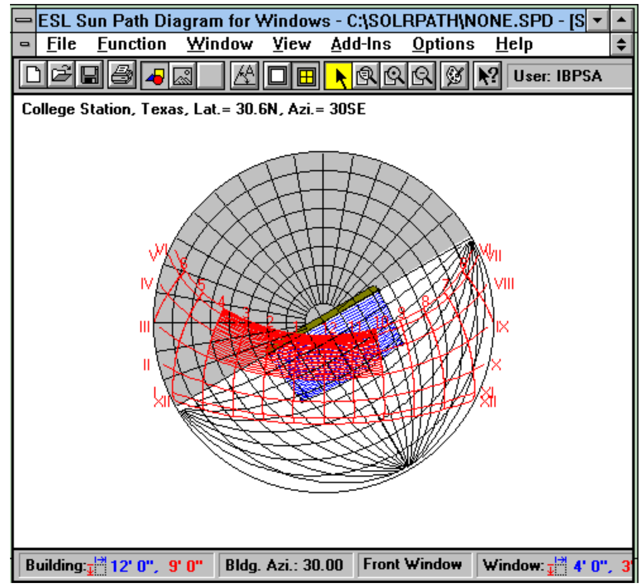

Figure 4: View of sunpath and shading from 2 ft. overhang. This sunpath diagram contains the previously described sunpath and the shading for a 2 ft. overhang that projects directly out from the top of the window.

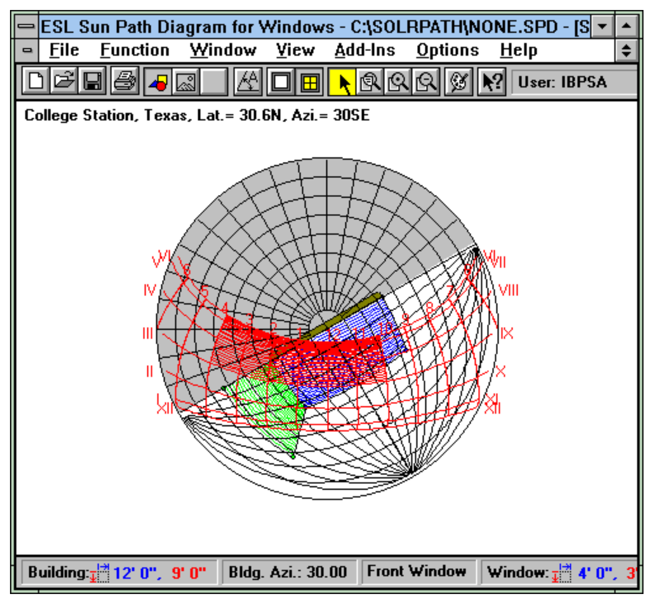

Figure 5: Sunpath diagram and shading from overhead shade and 2 ft by 2 ft right fin. This view shows the combined shading from the previously described overhead shade and a 2 ft. by 2 ft. fin placed directly below the overhead on the right side of the window (as the viewer faces out).

the window and is connected to the overhang along its upper edge. The effect of this shade can be seen in Figure 5. The shape and placement of this additional shading would also need to be calculated and placed on the shading mask protractor. One way to do this is with another arctangent relation that evaluates the 1 ft rise and 2 ft window width, or

$$\tan^{-1} c = \frac{H-2}{W/2} = \frac{1}{2} = 26.5^{o} \qquad (3)$$

The 26.5 degree angle is then used to determine the point that represents the interception of the bottom of the fin and the plane of the window. The shape of the fin can then be filled in using the appropriate shading mask construction lines that represent the lower edge of the right fin (i.e., line A’’-D in Figure 3) and the construction line that represents the vertical edge of the fin (line C-D in Figure 3).

Clearly, the manual method can produce an accurate assessment of a particular rectangular shading device. However, the amount of repeated calculation that is needed can tax the designer and can be error prone. The inconvenience of the traditional methods of photocopying the traditional sunpath diagrams can now be bypassed with the use of a new computer program. With this program the user only needs to enter the building latitude, along with dimensions of the window and shading devices if any, and the program produces the correct sunpath diagram and shading mask protractor. The remainder of this paper describes the use of the new computer program.

USING THE MS-WINDOWS BASED SHADING ANALYSIS PROGRAM

The sunpath diagram and shading mask protractor represents an ingenious graphical overlay that simplifies the 2-D display and understanding of two 3-D hemispherical coordinate systems. Mathematically, the shading mask protractor can be represented with three line types, namely, concentric circles, radial lines and two series of ellipses. The concentric circles represent the altitude of a point on a hemisphere above the horizon, the radial lines represent the edge of a vertical surface with respect to its off-south azimuth, and the two series of ellipses represent horizontal lines which run parallel or perpendicular to the plane of the window.

The second set of hemispherical coordinates represents the sun’s path across the sky dome for a given latitude. Such a path can be described with two additional sets of curves. The first set of curves represent seven traces of the sun across the sky from sunrise to sunset [2]. The second set of curves on the sunpath represent the location of the sun at the same time-of-day throughout the year[3]. The curves that represent the sun’s path can be described by selectively plotting the well-known solar equations that describe the sun’s zenith angle and the off-south solar azimuth angle (Duffie and Beckman 1991; Kreider and Kreith 1978). Additional details about the mathematics and algorithms that can be used to construct the shading mask protractor and sunpath diagram can be found in the references by McWatters and Haberl (1994a; 1994b; 1995).

Figure 6 shows the opening screen for the solar shading analysis program. The combined sunpath diagram and shading mask protractor can be seen in the upper right quadrant. All data entry is made in the “data input” screen to its left. Modifications to the shading devices are shown on the various views of the building and window in the lower screens. The effect of the shading is shown on the combined sunpath diagram and shading mask protractor. In Figure 6 the shading of a window that is 4 feet in width by 3 feet in height facing 30 degrees east of south at a northern latitude of 30.3 degrees is shown[4]. The shading devices, shown in Figure 6, consist of an overhang accompanied by left and right fins with the dimensions as shown in Figure 7. The coordinates for the windows are measured from a point centered at the bottom of the window. The x, y, z reference points locate a point in the center of each shading plane. The results of the shading analysis are shown in the accompanying sunpath diagram in the upper right of Figure 6. The time that has been targeted for shading (i.e., 10:00 a.m. to 4:00 p.m.) is entered into the data entry screen shown in Figure 8.

The solar shading analysis program can be used to represent any north or south latitude, including latitudes at or above the arctic circle, such as the 65 degree north

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[2] One trace each for December and June, and one trace for each pair of months between December and June, specifically; November-January, October-February, September-March, August-April, and July-May. The traces for December and June represent winter and summer solstice.

[3] This assumes solar time.

[4] The shading indicated in Figure 6 is color coded with a separate colors to match to match the horizontal shade, left and right fins. Separate shading is indicated for that area of the shading mask protractor that represents the area shaded by the vertical wall. The width of the window is also indicated on the diagram as bold line in the center of the shading mask. The sun’s path has been darkened from 10:00 a.m. until 4:00 p.m. from March 21st through September 21st (i.e., the summer cooling season).

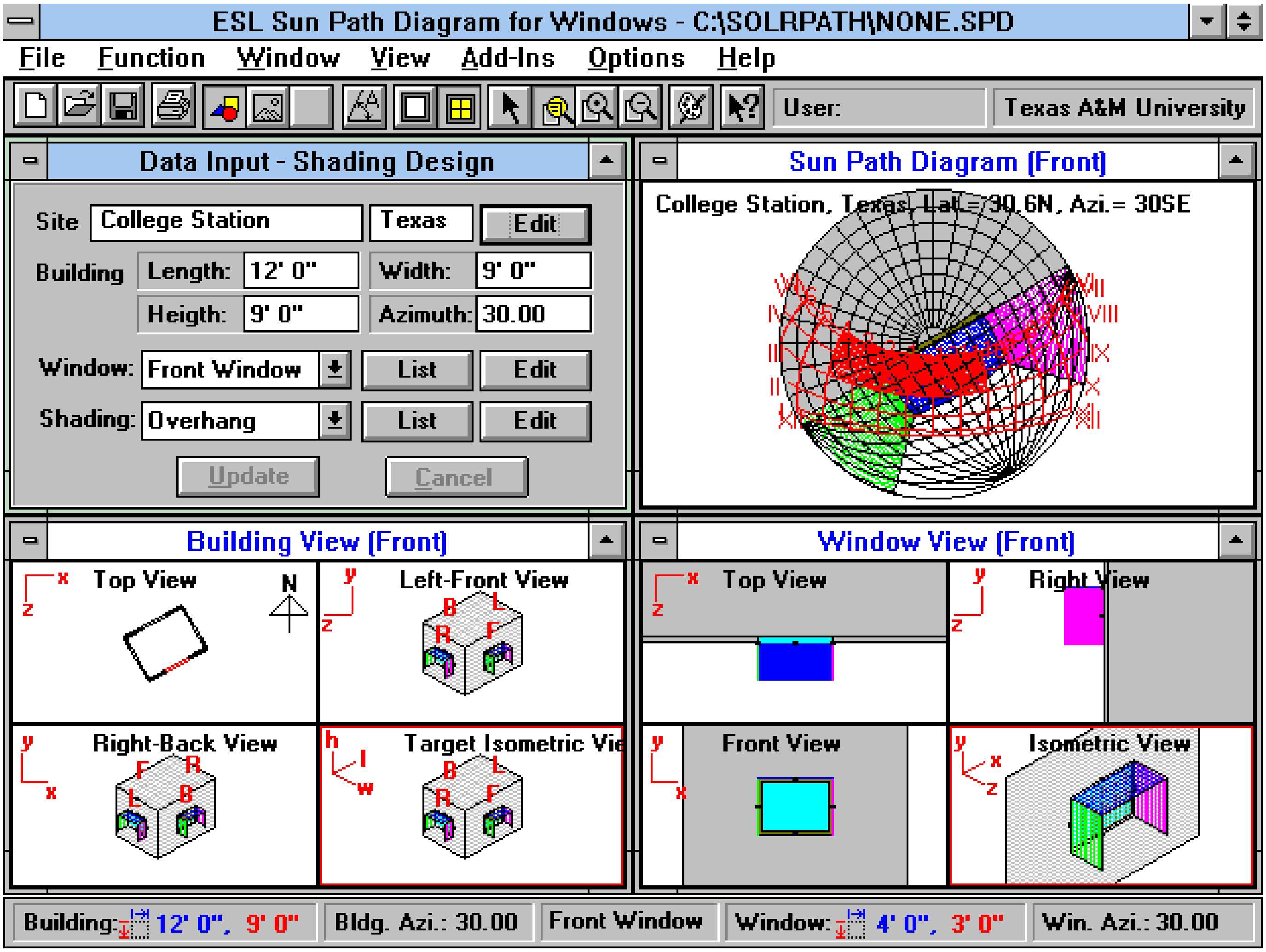

FIGURE 6: Opening screen for the MS Windows solar shading analysis program. This figure shows the opening screen for the sunpath shading analysis program. All data entry is made in the "data input" screen. Modifications to the shading devices are shown on the various views of the building and the effect of the shading is shown on the combined sunpath diagram and shading mask protractor.

latitude shown in Figure 9, or latitudes in the southern hemisphere, such as the 30 degree south latitude shown in Figure 10 in which the target shading time of 10:00 a.m. until 4:00 p.m. is indicated on the sun's path for the cooling period from September through March for a facade facing 30 degrees west of north. Obviously, summer shading in the southern hemisphere would need to be provided for a north, east or west facing facade.

SUMMARY

This paper has presented a new software package that has been developed to fast-track the learning and use of the sunpath diagram and shading mask protractor which is based on previously published equations for plotting the sunpath diagram and shading mask protractor. Examples were provided that compared the traditional method and the new software.

The use of the program at the university has proven to be extremely beneficial in conveying to the students the usefulness of the shading mask protractor and sunpath diagram. Future work includes: i) the development of an external shape shading functions that will allow one to analyze the effect of multiple objects placed in front of the window at various orientations (such as other buildings); ii) the addition of the ability to project hourly measured solar data upon the sunpath diagram and to modify it according the to incidence angle of the plane of the window, and iii) the addition of a thermal analysis to predict the heat gain through an actual window.

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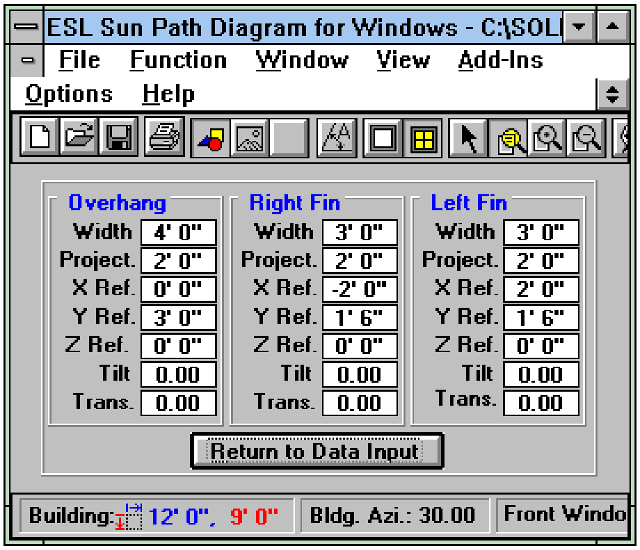

FIGURE 7: Data Input Summary Screen. This figure shows the data input screen where all pertinent data for the currently displayed facade are shown. The data shown represent the dimensions of the horizontal shade and left and right fins shown in Figure 6.

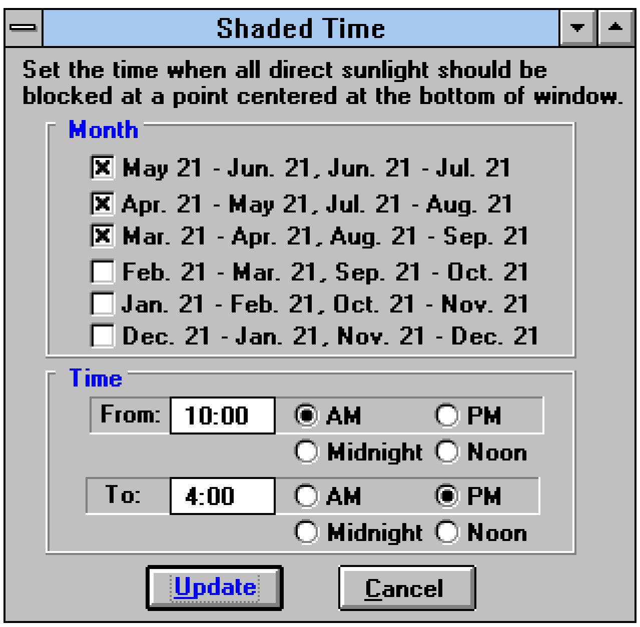

FIGURE 8: Solar Target Time. This figure shows the data input screen for the time of day, and time of year that shading is to be provided for Figure 6.

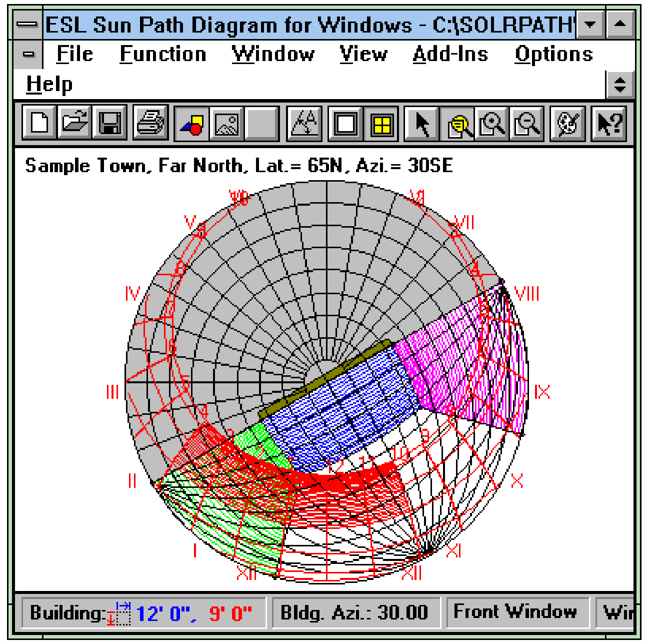

FIGURE 9: Sunpath Diagram and Shading Mask Protractor for 65 Degrees North Latitude. This combined diagram shows the sunpath diagram and shading mask protractor for a vertical surface facing 30 degrees east of south.

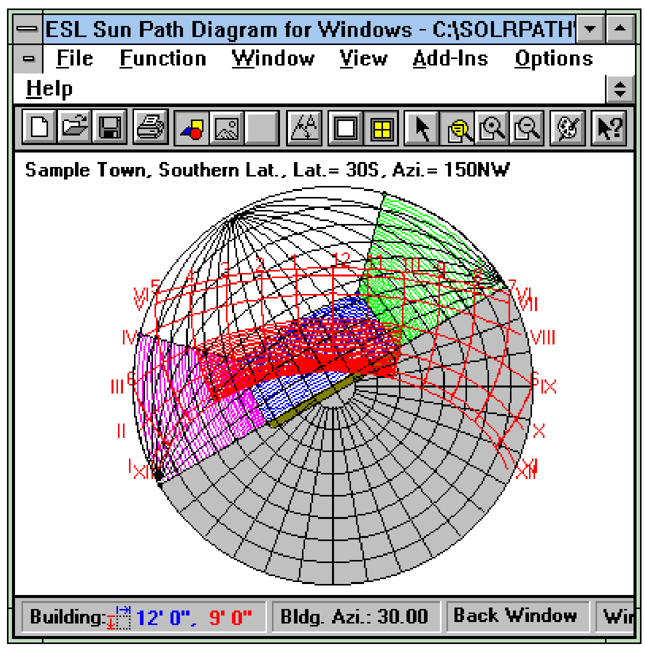

FIGURE 10: Sunpath Diagram and Shading Mask Protractor for 30 Degree South Latitude. This combined diagram shows the sunpath diagram and shading mask protractor for a vertical surface facing 30 degrees west of north for a period from September through March.