MOISTURE PERMEABILITY DATA PRESENTED AS A MATHEMATICAL FUNCTION APPLICABLE TO HEAT AND MOISTURE TRANSPORT MODELS

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ABSTRACT

The physics of moisture transfer is complex and as a result modelling is normally carried out on a macroscopic basis, with empirical coefficients used to simulate the transport process. These coefficients are not single values but depend on the moisture content of the material. The application of this data within the simulation environment requires the determination of a systematic methodology for its presentation. This paper investigates the transport coefficient referred to as the vapour (or moisture) permeability. This coefficient is central to the accurate simulation of moisture transport. A detailed analysis of various functional relationships between vapour (or moisture) permeability and relative humidity is carried out and a possible form of equation proposed. An initial data-base of material moisture permeability data is then reported for a range of common building materials.

INTRODUCTION

The need for computer simulations which can predict accurately the moisture behaviour of building structures is increasing as new construction materials and techniques become commonplace and improved energy efficency becomes a priority.

The main issue which inhibits progress in the development of moisture simulation models which can be used by designers is the lack of reliable experimental data upon which models can be based. This is related to both the complexity of arranging accurate and repeatable measurements and the slow speed of moisture movement through some porous materials which necesitates lengthy experimental periods. It is also the case that databases of material properties which are available often lack information on the test conditions or are in a form which is difficult to apply directly within prediction models.

This paper investigates one of the central moisture transport parameters, the vapour (or moisture) permeability, and proposes a method of data presentation which is in a form which would provide maximum material performance information. An initial data-base of moisture permeability coefficients is then provided based on experimental measurements carried out by the authors.

VAPOUR (OR MOISTURE) PERMEABILITY

The process of moisture transport through porous media is an extremely complex phenomenon with parallel/series moisture flow taking place in both the vapour and liquid phases[1]. The extent of liquid water present within a porous material is highly dependant on the pore size distribution, although it is recognised that, for most hygroscopic building materials, capillary condensation will be initiated at relative humidities of about 60%[2]. At such levels of relative humidity some pores will be completely liquid-filled, while others will be partially filled, with condensation/re-evaporation continuously occurring within the material voids. The transport of moisture in the vapour state as a pure Fickian diffusion process requires an absence of capillary condensation and is therefore confined to relative humidities below those normally encountered within building structures in winter in the UK.

It is very difficult, if not impossible, to experimentally separate moisture transported as liquid from that transported as vapour. The measurements which are carried out do, in fact, involve the determination of the total moisture flux through a sample under isothermal conditions[2]. This total flux will include moisture transport both in the liquid and the vapour state. The measured value normally termed in literature as ‘vapour permeability’ would, therefore, be more accurately defined as the ‘moisture permeability’ and will subsequently be referred to in this paper as such.

The total moisture transfer (j_{tot}) through a sample of material as a result of both liquid and vapour components under isothermal conditions can be expressed as [3][4]:

$$j_{tot} = j_v + j_l \quad (1)$$

where j_v = vapour flux (kg/m^2s)
j_l = liquid flux (kg/m^2s)

Equation (1) can be expanded to give:

$$j_{tot} = -\left(D_v + \frac{D_l R_v T \rho_l}{\phi p_s}\right)\nabla p_v \quad (2)$$

where D_v = diffusion coefficient of water vapour in air(m^2/s).
D_l = liquid permeability (s)
R_v = gas constant for water vapour, 461.52 (J/kgK)
T = absolute temperature (K)
ρ_l = water density (kg/m^3)
p_v = vapour pressure (N/m^2)
p_s = saturation vapour pressure (N/m^2)
ϕ = relative humidity, p_v / p_s

Equation (2) can also be expressed as:

$$j_{tot} = -\mu \nabla p_v \quad (3)$$

where μ could be described as the 'differential moisture permeability' as it relates the total flux of moisture to the pressure gradient across a differential thickness of material. The differential permeability (μ) is a function of the relative humidity which for hygroscopic materials is highly non linear.

For a finite thickness of material l, the integrated form of equation (3) can be written as

$$j_{tot} = \frac{\bar{\mu}\,(p_{v1} - p_{v2})}{l} \quad (4)$$

where p_{v1} and p_{v2} are the vapour pressures at either boundary of the material and the coefficient $\bar{\mu}$ is the 'average moisture permeability' pertaining to the varying humidity conditions along the moisture transmission pathway l. $\bar{\mu}$ is related to the differential permeability by

$$\bar{\mu} = \frac{1}{(\phi_2 - \phi_1)} \int_{\phi_1}^{\phi_2} \mu \, d\phi \quad (5)$$

where ϕ_1 and ϕ_2 are the boundary relative humidities.

Equation (4) forms the basis for a large number of the dynamic models currently available for moisture transmission simulation.

THE MEASUREMENT OF MOISTURE PERMEABILITY

The measurement methods currently adopted for moisture permeability normally involve the gravimetric determination of the total moisture flow rate for a known vapour pressure difference maintained across a test sample under isothermal conditions[2][5].

The material to be tested is sealed into the mouth of an impermeable cup containing a vapour pressure regulator which can be water, dessicant or a saturated salt solution. The cup is then placed in an environmental chamber and a constant vapour pressure difference is maintained across the sample. Once equilibrium is attained, the vapour flow can be determined from the steady change in cup weight with time. The average moisture permeability $\bar{\mu}$ for the experiment is then calculated from equation (4), where p_{v1} and p_{v2} now represent the vapour pressures inside and outside the cup.

This approach is employed by the relevant authorities in most countries, although there are differences in the recommendations given for the cup design, the vapour pressure regulators and the environmental chamber conditions[5].

Values of $\bar{\mu}$ obtained from standard cup tests as described above are generally unique to the conditions of the test and cannot be extrapolated to other conditions. The prediction of $\bar{\mu}$ for any given set of boundary conditions, which constitutes one of the central prerequisites for application of modelling techniques, requires the specification of the full functional relationship $\mu(\phi)$ together with the application of equation (5).

The differential permeability cannot be determined directly but it is possible to generate the function $\mu(\phi)$ from values of $\bar{\mu}$ measured from a series of cup tests covering the whole humidity range. This involves the use of a variety of vapour pressure regulators and/or chamber conditions[2][5].

THE DETERMINATION OF AN APPROPRIATE $\mu(\phi)$ RELATIONSHIP

The form of functional relationship $\mu(\phi)$ chosen should be able to be applied to all construction materials. This would simplify the analysis and presentation of measurement data by providing a common mathematical approach. A number of researchers have presented moisture permeability data in literature using a variety of permeability/relative humidity relationships as indicated in Table 1.

Table 1 Proposed Differential Moisture Permeability Relationships

Eqn. No.	Author	Materials Reported	Form of Equation
1	Various [2][6]	concrete, brick, cement mortar, polystyrene, concrete, sand lime, stone, fibre cement	A+Bexp(Cϕ) [ϕ as a %]
2	Burch [6]	plasterboard	exp(A+Bϕ) [ϕ as a %]
3	Galbraith, McLean [4][7]	particle board, plywood, wood, brick, plasterboard, extruded polystyrene, expanded polystyrene, concrete	$A + B\,\phi^{C}$ [ϕ as a fraction]
4	Richards [6]	particle board	$\exp(A+B\phi+C\phi^2)$ [ϕ as a %]

It is important to note that the relative humidity (ϕ) in equation types 1, 2 and 4 is given as a percentage while for equation 3 it is given as a fraction. In order to investigate the suitability of each relationship, it is necessary to compare the efficacy of each equation type to accurately represent real experimental data. Such comparisons require two principal criteria. First, there is the accuracy with which each equation fits the experimental data. Second, in all cases, each equation must exhibit characteristics which conform to the accepted physical principles of moisture transfer.

Table 2 Moisture Permeability Measurement Data

Material	Test Temp. (°C)	Test Relative Humidity (%) (side 1/side 2)	Permeability (kg/msPa) x 10^{-11}
Plasterboard [6]	23	0 / 28.7	3.30
		0 / 50.9	3.00
		22.7 / 50.8	2.50
		43.2 / 61.5	2.50
		50.9 / 69.1	3.80
		61.5 / 81.2	4.20
		61.4 / 100	4.00
		80.4 / 100	5.20
		92.4 / 100	6.50
Plywood [6]	not stated	0 / 50	0.027
		0 / 76.6	0.058
		50 / 100	0.500
		76.3 / 100	0.980
Expanded Polyurethane [7]	23	0 / 60	0.358
		60 / 80	0.422
		60 / 93	0.488
		60 / 100	0.503

For example, at relative humidities below about 60%, the permeability should be relatively constant, being representative of a pure vapour transfer process.

A data-set of average permeability values for three common building materials is given in Table 2. These materials include plaster board which is relatively non-hygroscopic, plywood to represent wood - based materials and expanded polystyrene as an insulation material. The source of the data given is from measurements made by the authors[7] and from the IEA Annex 24 Database of Material Properties which is currently accepted as one of the most comprehensive collections of moisture permeability data compiled to date[6].

Equation Coefficients

The data-set of experimental measurements given in Table 2 was used to determine the best-fit equation coefficients for each of the proposed mathematical relationships for differential permeability. This involved the integration of the equations to produce a mathematical expression for the mean permeability, $\bar{\mu}$, in terms of the relative humidity limits on each side of the test sample. The integrated form of each equation was then used along with the experimental data of mean permeabilities to determine the coefficients which give a best-fit equation to the experimental data.

Table 3 Calculated Curve Fit Coefficients

Eqn. No.	Material	Equation Coefficients A	B	C	Mean r^2
1	Plaster Board	2.87E-11	9.60E-14	6.1E-2	
	Plywood	3.13E-13	1.60E-16	1.2E-1	
	Exp. Polyurethane	3.41E-12	2.36E-14	5.1E-2	0.951
2	Plaster Board	6.53E-1	1.10E-2	--	
	Plywood	-8.070	8.90E-2	--	
	Exp. Polyurethane	-1.261	6.70E-3	--	0.871
3	Plaster Board	2.91E-11	3.99E-11	4.931	
	Plywood	3.20E-13	2.55E-11	9.770	
	Exp. Polyurethane	3.50E-12	3.73E-12	4.510	0.953
4	Plaster Board	1.326	-1.70E-2	2.3E-4	
	Plywood	-1.460	-9.39E-2	1.2E-3	
	Exp. Polyurethane	-8.83E-1	-1.18E-2	1.7E-4	0.957

Non-linear regression techniques, supplied by the Statistical Package For The Social Scientist (SPSS) computer programme Version 6, were used to provide curve fitting for the differential permeability (μ) against relative humidity (ϕ) for each proposed equation and investigated material. The results of this curve fitting exercise are given in Table 3, where A, B and C are the best-fit equation coefficients obtained from the non-linear regression and r^2 is the Coefficient of Determination, the value of which gives an indication of the goodness of fit of each proposed equation.

Suitability of Equations

Table 3 indicates that equations type 1, 3 and 4 produce mean r^2 values which are above 0.9 while equation type 2 has a notably lower mean r^2 value of 0.871 for the three materials tested.

The resulting differential permeability functions for each material for equation types 1, 3 and 4 are presented as Figures 1,2 and 3 respectively. The following observations can be made.

Figure 1 Differential permeability curves obtained from equation 1

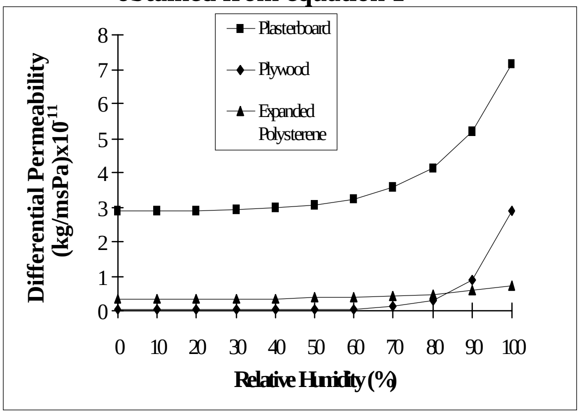
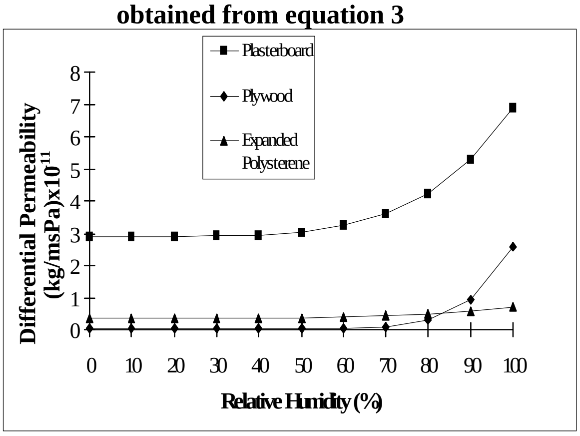


Figure 2 Differential permeability curves obtained from equation 3



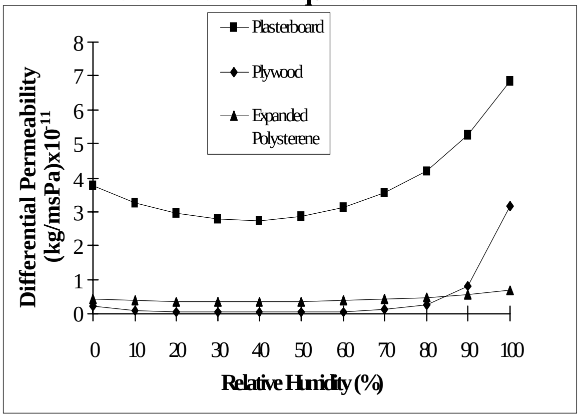
The differential permeability curves for equation type 4 (Figure 3) clearly exhibit a different shape from that produced by equations 1 and 3.

Equation type 4 exhibits a decrease in differential permeability with increasing relative humidity up to 40%. This is not consistent with the process of vapour diffusion which would be the dominant transfer mechanism in this humidity range.

Equation types 1 and 3 produce differential permeability curves which are consistent with the current understanding of the physics of moisture transport. They also have high r^2 values indicating a good fit with the experimental data.

Clearly, on the basis of the three materials tested, there is little to choose between equation types 1 and 3 and both could be accepted as the basis for a standardised system of data presentation.

Figure 3 Differential permeability curves obtained from equation 4



AN INITIAL DATA-SET OF MOISTURE PERMEABILITY COEFFICIENTS

The authors have carried out over the last decade a large volume of moisture permeability measurements using experimental apparatus which conforms to the requirements of the relevant British Standard (BS 4370, Part 2, 1973). These experiments have involved measurements, for each material, of the average moisture permeability for at least four sets of relative humidity conditions covering the full range of humidity conditions. Table 4 gives the measurement data obtained for six additional building materials along with the predicted permeability values obtained using an equation type 3 curve fit. The measurement data presented is the mean permeability obtained from at least five individual cup measurement on each material at each test condition.

The curve fit coefficients for each material are given in Table 5. This table includes the curve fit coefficients for the data in Table 2. These curve fit coefficients represent a data base of differential permeability curves for a total of ten common building materials in a form which can be applied in a simulation model. The curve-fit coefficients can be used to determine the average permeability for any set of conditions as:

$$\bar{\mu} = A + \frac{B}{(\phi_2 - \phi_1)[C+1]}[\phi_2^{c+1} - \phi_1^{c+1}] \qquad (6)$$

The total moisture flow rate can then be determined using equation 4.

The curve-fit coefficients in Table 5 have been applied to equation 6 to predict average permeability values for the specified test humidity ranges. These are shown in Table 4 along with the measured values and a percentage error calculated for each.

Table 4 Measurement Data for Common Building Materials

Test Material	RH Range %	Measured $\bar{\mu}$ x10[12]	Predicted $\bar{\mu}$ x10[12]	% Difference
Brick (Clay)	0-60	11.29	11.20	-0.8
	80-60	12.11	12.50	+3.2
	93-60	14.50	13.90	-4.1
	100-60	14.76	15.00	+1.6
Norwegian Spruce	0-60	1.6	1.6	0.0
	90-60	9.5	9.41	-0.9
	100-60	12.8	13.0	1.6
	100-80	20.7	20.6	-0.5
Aerated Concrete (490 kg/m^3)	0-60	21.56	21.60	+0.2
	80-60	25.01	24.70	-1.2
	93-60	28.74	29.10	+1.2
	100-60	33.45	33.30	-0.4
Aggregate Concrete (1327 kg/m^3)	0-60	12.26	11.70	-4.6
	80-60	11.40	12.70	+11.4
	93-60	16.62	15.50	-6.7
	100-60	18.66	19.00	+1.8
Particle Board	0-60	3.66	3.56	-2.7
	80-60	3.89	4.17	7.2
	93-60	5.59	5.30	-5.2
	100-60	6.43	6.54	1.7
Extruded Polystyrene	0-50	2.28	2.28	0.0
	80-60	2.11	2.28	0.8
	93-60	2.50	2.28	-8.8
	100-60	2.12	2.28	7.5

Table 5 Data Base of Moisture Permeability Coefficients

Test Material	Equation Type 3 Curve-fit Coefficients		
	A	**B**	**C**
Brick	1.11×10^{-11}	1.16×10^{-11}	6.21
Norwegian Spruce	1.40×10^{-12}	3.55×10^{-11}	6.51
Plywood	3.20×10^{-13}	2.55×10^{-11}	9.77
Plasterboard	2.91×10^{-11}	3.99×10^{-11}	4.931
Aerated Concrete	2.15×10^{-11}	4.11×10^{-11}	7.62
Aggregated Concrete	1.16×10^{-11}	3.43×10^{-11}	10.54
Particle Board	3.55×10^{-12}	1.20×10^{-11}	8.91
Expanded Polyurethane	3.50×10^{-12}	3.73×10^{-12}	4.51
Extruded Polystyrene	2.28×10^{-12}	0.00	1.00

Inspection of Table 4 reveals that for Brick, Norwegian Spruce and Aerated Concrete the deviations in predicted average permeability from the measured values are in each case less than 5%. The highest deviation is for Aggregated Concrete (+11.4% at 80-60% relative Humidity). The remaining materials (Particle board and Extruded Polystyrene) have in all cases deviations of less than 10%.

CONCLUSIONS

The four equations investigated in this paper have been used by researchers to provide mathematical functions which adequately fit the experimental data which they have obtained for the particular materials which they have investigated. In the wider context it is important to have a generalised form of equation which will give an adequate and realistic curve-fit for the full range of materials used in construction.

Equation type 1, $\mu = A + B \exp(C\phi)$, and equation type 3, $\mu = A + B\,\phi^C$, both yielded differential permeability functions which provide a good fit to the experimental data obtained for the three representative building materials and are consistent with the accepted physical principles of moisture transport in porous media. However, equation 3 produced a slightly better fit to the experimental data having a higher mean r^2 value.

The suitability of equation 3 has been demonstrated by its application to measurement data covering an additional six common building materials. The experimental data as well as the resulting moisture permeability coefficients are given in a form which could be directly applied within moisture simulation models.

The accuracy of the predictions obtained is shown to be very favourable and indicates that the use of a generalised form of differential permeability equation is a realistic approach to the presentation of moisture permeability data.

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