SIMULATION AND FAULT DETECTION OF THE THERMAL STORAGE SYSTEM

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ABSTRACT

The control strategy of the thermal storage HVAC system gives a large effect to the storage efficiency which dominates the tank volume to a great extent. Authors introduce how the temperature distribution of the tank varies and gives a considerable damage on the HVAC system performance, and how this kind of fault can be detected and diagnosed through the pattern recognition of temperature profiles which are obtained by storage system simulations.

INTRODUCTION

The water and/or ice storage system is widely used in Japan for the power security. There are two typical types used for the water storage, one is the so-called multi-connected complete mixing tanks and the other is the single temperature stratified tank. Several design factors contribute to the size of the tank. Nakahara et al. [1][2] showed how to decide the storage efficiency and identified the significant factors and sensitivity of them. The system control strategies are the principal factors to keep high efficiency, which would give some vital effects when they become malfunctioned.

Then, how to detect and diagnose these malfunctions becomes an important subject. Authors proposed to use the pattern recognition method using the temperature profiles in case of the normal and faulty conditions for the most popular type of the tank, that is, the multi-connected complete mixing tanks at the IEA Annex25 research project [3]. The present paper introduces that precisely the same approach is available for another typical tank of type, that is, the single tank with the temperature stratification.

STORAGE EFFICIENCY

From the practical design point of view, the storage efficiency η_s, which is applied to calculate the tank volume shown in Equation (1), is defined as **Figure 1** using the temperature profile.

$$V = Q_{st} / \rho c \eta_s \Delta\theta_0 \qquad (1)$$

Where, in Eq (1) and Figure 1,

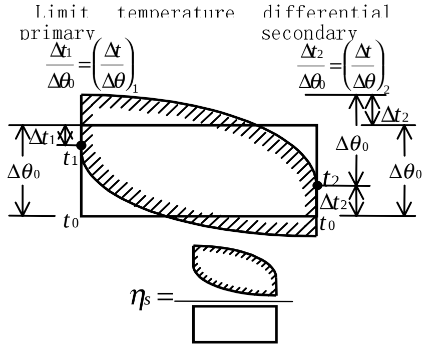


Figure 1 Definition of the storage efficiency

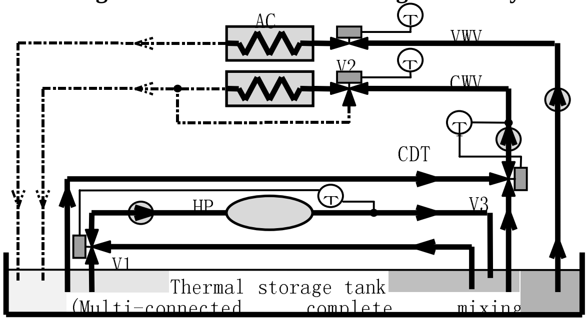


Figure 2 Simplified diagram of thermal storage system

$\boldsymbol{V}$: water volume, $\boldsymbol{Q_{st}}$: heat to be stored, $\boldsymbol{\Delta\theta_0}$**:** reference temperature difference of the chilled water, defined as the weighted coil design temperature difference at the peak load, $\boldsymbol{t_0}$: set point outlet temperature of HP, the heat pump, $\boldsymbol{t_2}$: allowable highest temperature of the chilled water to the cooling coil, $\boldsymbol{t_1}$: allowable lowest temperature to HP, $\boldsymbol{\Delta t_2}$: limit temperature differntial= $\boldsymbol{t_2}$-$\boldsymbol{t_0}$, $\boldsymbol{\Delta t_1}$: limit temperature differential=$\boldsymbol{t_0}+\Delta\theta_0-\boldsymbol{t_1}$, ρ **:** density, $\boldsymbol{c}$:specific heat

The horizontal axis of Figure 1 is the position, or the volume, of the tank *V*. Thus, the storage efficiency is the ratio of the actually stored heat to the nominal heat stored in the water volume ***V*** with the reference temperature difference of $\Delta\theta_0$. Further discussion on the storage efficiency shall be referred to [1].

SIMPLIFIED SYSTEM DIAGRAM

The **Figure 2** shows a simplified diagram of HVAC system with the thermal storage. What are important from the viewpoint of the storage efficiency are as follows. These are significant factors of the storage efficiency as shown in the next chapter and often malfunction, which will give unfavorable damage.

1)The VWV, or the variable water volume system, controlled by the two-way valve V2 and the CWV, or the constant water volume system, controlled by the three-way valve intermingle.

2) The three way mixing valve V1 controlled in PID action is used at the suction line of the heat pump, HP, in order to keep the set-point temperature of the outlet water constant. It keeps the water temperature to the cooling coil low enough during the daily cycle.

3) Another three-way control valve V3 in PID action may be used to keep the return water temperature from the cooling coil as high as possible for the CWV system. This control loop is called as CDT, hereafter.

4) With this system composition, the water in the tank turns around only once a day.

5) In addition, there are several more factors which contribute to keep high storage efficiency such as the the difference between the HP outlet temperature, the allowable highest temperature rise, etc. [1]

ESTIMATION TABLES OF STORAGE EFFICIENCY

Simulations were carried out for eighty-one cases, the combination of which was determined by the design of experiment. Two days of simulation was supposed enough to determine the steady state and the limit temperature rise of the lowest temperature tank is examined if it reached the value just as predetermined [1]. The 1-D finite difference method was applied to calculate the temperature distribution. Reference [2] shows the detailed modeling of the temperature stratified tank. As to the system components, only the cooling coil was precisely modelled, because the outlet temperature of the coil is the most important.

The significant factors and interactions between two factors were abstracted through the analysis of variance and the estimation tables of the storage efficiency were obtained for two types of the thermal storage tank. The detail of these process, together with description of the mixing models and consideration on how the factors and levels were selected, are described elsewhere.[1][2]

The **Table 1** shows the factors and levels chosen for the simulation and statistical analyses. The **Table 2** is the estimation tables of storage efficiency for two types of tanks composed of significant factors. $\varDelta_{ij}$ is the factor effect to be summed up over all significant factors. When actual temperature difference is kept higher than the designed due to high temperature return water at the partial load of VWV system, the storage efficiency may become larger than 100%.

Table 1 Factors and levels for experimental design

Factor (i)		Level (j)		
i	factor name	1	2	3
A	Peak dimensionless depth of complete mixing region	0.1	0.3	0.7
B	Min./Max. Load ratio	0.8	0.5	0.2
C	Existence of CDT for CWV	Yes(2)	Yes(1)	No
D	CWV Load ratio to total load	0.2	0.5	0.8
E	Limit temp. difference ratio for primary circuit	0.6	0.4	0.2
F	Limit temp. difference ratio for secondary circuit	0.4	0.3	0.2
G	Number of tanks	40	20	10
H	Operation schedule of generator	0-24	18-12	22-8

Table 2 Estimation table of the thermal storage efficiency

TYPES		**Multi-connected complete mixing tanks**			**Single temperature stratified tank**		
	Factor (*i*)	$n_s = 96.41 + \Sigma\Delta n_{ij}$(%) Level (*j*)			$n_s = 90.92 + \Sigma\Delta n_{ij}$(%) Level (*j*)		
		1	2	3	1	2	3
$\varDelta\eta ij$	A	-	-	-	7.56	7.48	-15.03
	B	2.85	0.94	-3.79	2.07	-0.04	-2.03
	C	9.47	1.76	-11.23	9.67	1.27	-10.94
	D	3.63	0.20	-3.83	2.79	-0.12	-2.67
	E	-	-	-	4.07	0.72	-4.80
	F	7.19	0.76	-7.95	6.91	-1.58	-5.33
	G	6.20	-0.21	-5.98	-*1	-*1	-*1
	H	5.50	-7.29	1.79	6.83	-9.17	2.33
$\varDelta\eta ij\times\varDelta\eta i'j'$	B×D	-5.92	0.18	5.73	-4.41	0.21	4.20
		-0.30	-0.49	0.79	-0.75	0.35	0.39
		6.22	0.31	-6.52	5.16	-0.56	-4.59
	C×D	-5.03	-0.32	5.35	-6.56	0.49	6.06
		-1.06	-0.23	1.29	0.25	-1.77	1.53
		6.09	0.54	-6.64	6.31	1.28	-7.59
	C×F	1.93	-1.29	-0.64			
		2.49	0.35	-2.84			
		-4.42	0.94	3.48			
	F×G	-1.01	-3.05	4.06			
		0.96	1.03	-1.99			
		0.05	2.01	-2.07			

*1 Number of tank = 1

Combination of levels($j\times j'$)

1×1	1×2	1×3
2×1	2×2	2×3
3×1	3×2	3×3

FAULT SIMULATION

The faulty phenomena observed in the thermal storage tank are affected not only by its proper faults but also by faults at any part of HVAC system. At the same time the thermal storage faults affect the operational performance of HVAC system and the building environment. Experiences show that a peculiar control fault around the thermal storage tank gives a characteristic transition of the temperature profiles of the tank, which somehow provides an information on the existence of some kind of faults.

The original simulation program was developed for calculating optimal storage capacity based on the given design conditions, so that it did not include the room model, and the room temperature was supposed to be kept constant. It is correct as long as the water temperature for cooling is kept proper due to the premise of optimal design with no-fault operation. In order to make it possible to simulate the faulty condition, where the water temperature is expected to reach abnormal condition in which the heating and/or cooling load to keep nearly constant room temperature cannot be extracted sufficiently, a simplified calculation model to estimate dynamic thermal behavior of the room was introduced [4].

After an optimal capacity of the tank is decided, two kinds of designated faulty conditions are given at the initial stage of simulation, that is at 0:00 in the morning, and the fault simulation begins only for one day for both conditions. The calculation results for each case are listed as follows.

a)optimal capacity
b)storage efficiency
c)temperature profiles for the normal condition
d)temperature profiles for two faulty conditions
e)room temperature variation and the rate of heat extraction
f)inlet and outlet water temperatures over the heat pump and the cooling coil

FAULTY CONDITIONS

The more a factor has a significant factorial effect, the more significant effects result when the factor goes out of order. The present paper pays attention to the importance of control strategies, because they have strong relations with the effects of these significant factors. Referring Figure.2 and Table.2, two kinds of faulty conditions are selected for examination.

1) The valve V1 malfunctions to fix its position as sucking all of water from the last tank to which chilled water returns from AHU coils and no water flow from the initial tank to the valve. This fault is the typical and most popular control fault in the primary circuit and shall be called as Fault ① in the following.

2) Both the valve V2 for VWV system and V3, when it is provided, for the CWV system went out of order or installed another way by mistake. It was supposed, as the result, that the two way valve worked as if it was a three way valve and that the V3 did not work to bypass flow from the last tank. This may be the typical and popular fault in the secondary circuit and called as Fault ② in the following chapters.

The reason why these faults are typical and popular is as follows. The HVAC is one of the most complicated and difficult system to control properly because of the large and frequent thermal disturbances, whereas the allowance for the environmental quality is not so narrow due to the flexibility of human comfort.On the other hand the HVAC designers have not sufficient knowledge on the control theory, while the control personnel, in the design as well as in the field , have little knowledge on the HVAC theory. The owners, in addition, care little for the maintenance quality.

That means no one cares how to tune the control parameters optimally, which results either hunting of actuators or widening the proportional band only for obtaining stability. Maintenance engineer, who is also knows little about relating knowledge, often manually forces to fix the control action, if ever no claim is fed back from occupants.

SIMULATION RESULTS

The temperature profiles as well as the heat balance between the rate of heat extraction and chiller output obtained by fault simulations in both types of storage tanks, for two cases among eighty one cases, are shown in **Figure 3 and Figure 4** as ① and ③ , for the normal and two faulty conditions. The figure ③ shows the cooling load, which has three patterns as shown in Table 1 by the B and D factors, and the heat pump operation profiles, which also has three patterns as shown in the same table by the factor H.

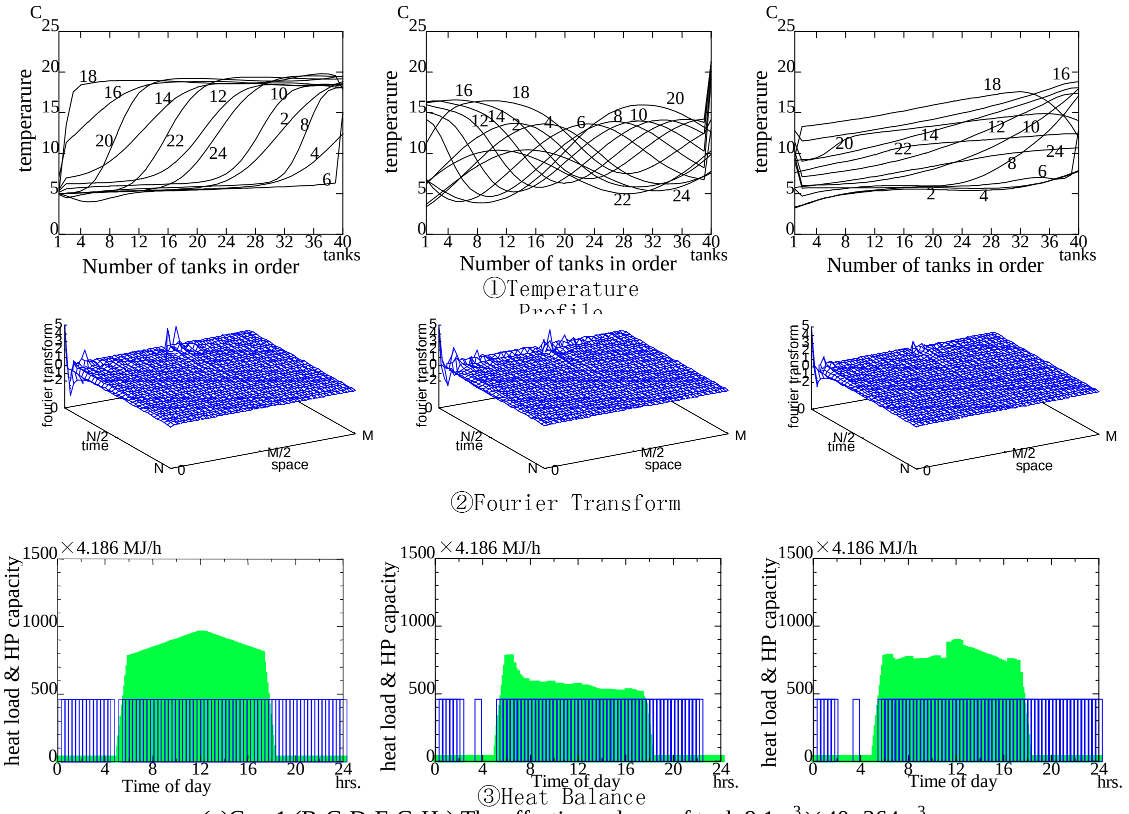
The temperature profiles are shown in two dimensions, the parameter of which is the time increment in every two hours. The most symptomatic phenomena for each fault are observed as follows. As seen in the heat balance graph, cooling load cannot be sufficiently extracted by the cooling coil in case the chilled water outlet temperature is raised abnormally higher due to the faults.

Normal operation

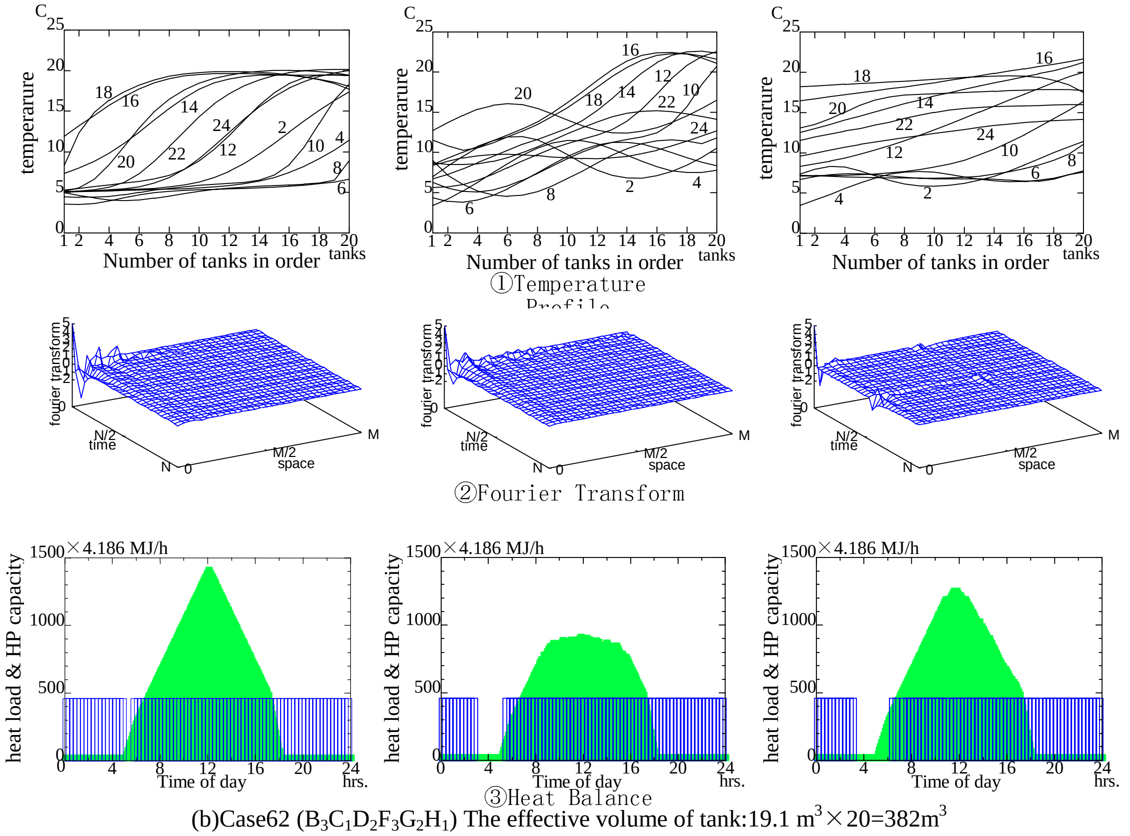
Watching carefully at the profiles, very few temperature profiles intersect each other in the normal operation. The peak temperature point of each profile is fixed at the position of the return tank, or the last tank. The highest temperature of the initial tank during a day is limited at the point of limited temperature rise following the design conditions.

Faulty operation ①

In the fault① condition the profiles wave and the peak temperature points gradually move,so that the profiles intersects each other.Thus, the initial tank temperature rises as the peak temperature point moves over the design condition. These symptoms do not always appear very clearly. When the design temperature difference for the primary, or the chiller, side, and the



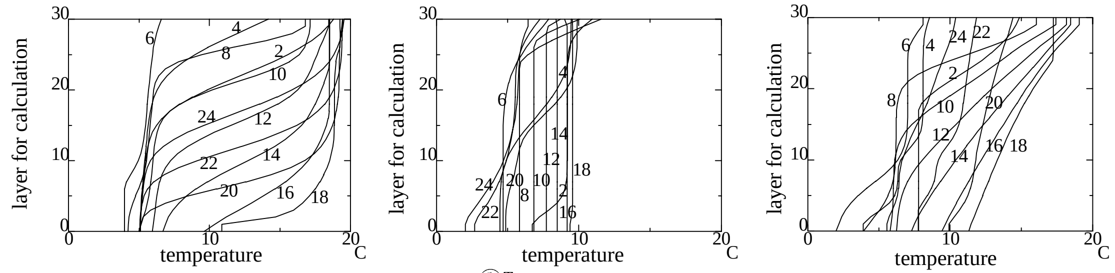
(a)Case1 ($B_1C_1D_1F_1G_1H_1$) The effective volume of tank:9.1m^3×40=364m^3



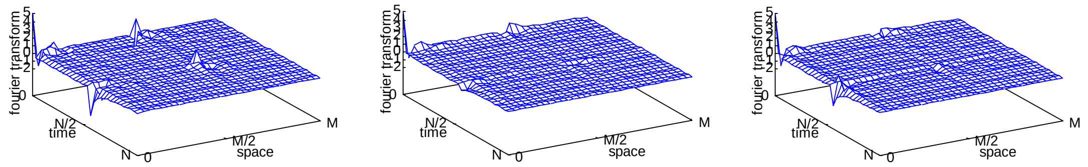
(b)Case62 ($B_3C_1D_2F_3G_2H_1$) The effective volume of tank:19.1 m^3×20=382m^3

Normal cases **Fault ① cases** **Fault ② cases**

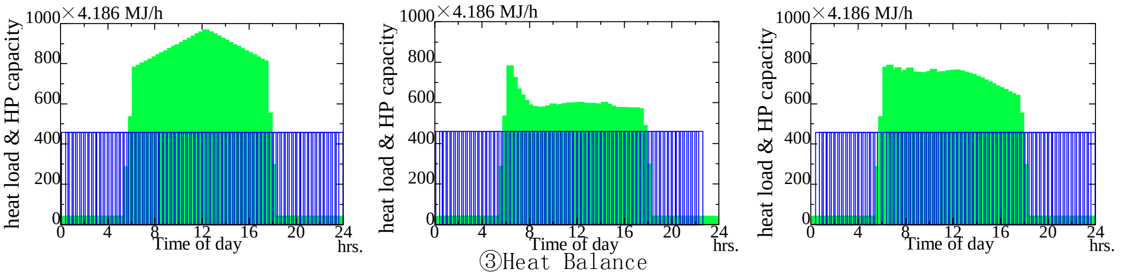
Figure 3 Temperature profiles and heat balance for the multi-connected complete mixing type tanks



①Temperature Profile

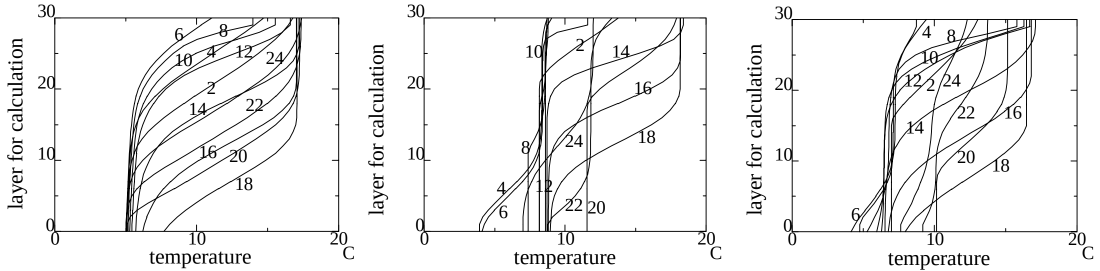


②Fourier Transform

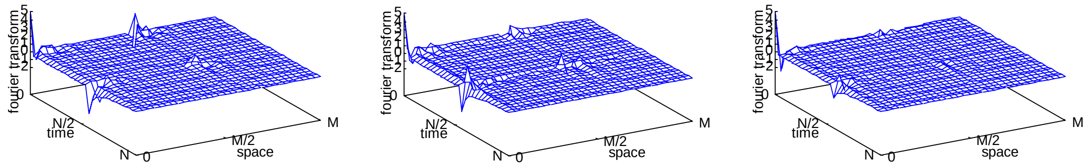


③Heat Balance

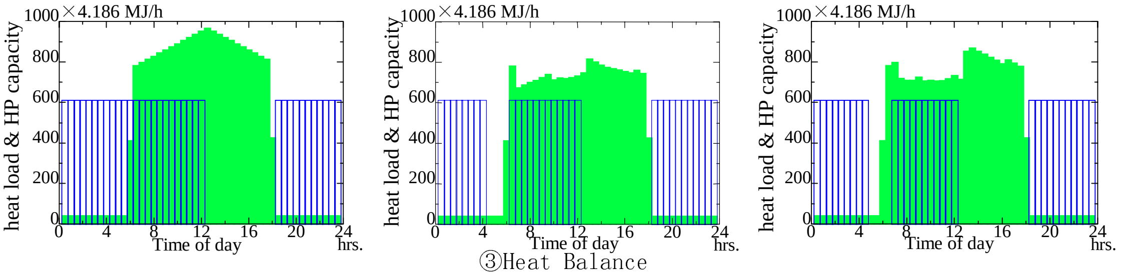
(a)Case1 ($A_1B_1C_1D_1E_1F_1H_1$) The effective volume of a tank:370.477 m^3



①Temperature Profile



②Fourier Transform



③Heat Balance

(b)Case27 ($A_1B_1C_3D_1E_3F_3H_2$) The effective volume of a tank:764.595 m^3

Normal cases **Fault ① cases** **Fault ② cases**

Figure 4 Temperature profiles and heat balance for the single temperature stratified tank

secondary, or AHU, side, are similar, the symptom becomes unclear. So is the results when the CWV system load overwhelms the VWV system load.

Moreover, if the temperature rise in the initial tank appears in the evening when the cooling load profile is much alike triangle, that means small cooling load in the early morning and late evening, the faulty phenomena does not affect the room temperature as long as it is the first day of the faulty operation. If the fault is not detected and recovered, the room condition will be vital in the following days.

Faulty operation ②

In the fault ② condition the profiles become rather flat, resulting the temperature rise over the designated limited value in the initial tank. This is because the elevated temperature of return water at the partial load, higher than the designed value, which is a typical phenomena of the VWV control using two way valve, does not appear. Again, it will be obvious that the phenomena will not appear clearly if the cooling load has a flat pattern, as the coil outlet water temperature is kept almost constant.

FOURIER TRANSFORMATION

As seen in of Figure 3 and 4①, the characteristics of the frequency and phase of the temperature profile pattern seem to have decisive factors to discriminate a fault from the other or from the normal. In order to quantify them the temperature profiles were analyzed into Fourier transformation. Two dimensional Fourier transform is expressed as Equation (2).

$$\begin{aligned}
[F_{kl}] &= \begin{bmatrix} C \\ S \end{bmatrix} \begin{bmatrix} C \\ S \end{bmatrix}_{kl} = \begin{bmatrix} C_k C_l & C_k S_l \\ S_k C_l & S_k S_l \end{bmatrix} \\
&= \frac{a_1 a_2}{NM} \sum_{r=0}^{N-1} \sum_{s=0}^{M-1} x_{rs} \begin{bmatrix} \cos \dfrac{2\pi k_r}{N} \\ \sin \dfrac{2\pi k_r}{M} \end{bmatrix} \begin{bmatrix} \cos \dfrac{2\pi l_s}{N} \\ \sin \dfrac{2\pi l_s}{M} \end{bmatrix} \\
& k = 0,1,2 \cdot\cdot N/2 \qquad l = 0,1,2 \cdot\cdot M/2 \\
& a_1 = \begin{cases} 2 & 0 < k < N/2 \\ 1 & k = 0, \quad N/2 \end{cases} \\
& a_2 = \begin{cases} 2 & 0 < l < M/2 \\ 1 & l = 0, \quad M/2 \end{cases}
\end{aligned} \tag{2}$$

Where,

x: temperature of the tank water [C], and it was supposed to be two dimensional real periodic function with the time and divided number as variables. Here, the divided number is the number of tanks in case of the multi-connected complete mixing tanks type, while it is the number of layers for numerical calculation in case of single temperature stratified tank type, ***C*,*S*:** sine and cosine components of Fourier transform, respectively, *N*, *M*: total time and total number of divided number of the tank , that is, the space.

The results [4] showed that composition of both the sine and cosine components are different between the normal the faulty for the identical cases but that the difference is sometimes not so large. The middle figure ② in Fig.3 and 4 shows the results of Fourier analysis. The difference mainly exists in the value, distribution and the degree of the sine and cosine components. This suggested a possibility of quantification using these factors as parameters for FDD.

PARAMETERS FOR MAPPING DATA

As shown above, factors describing the variation of the frequency and phase of the temperature profiles of the thermal storage tank are the value of sine and cosine components, their distribution and degree of the frequency of Fourier transform. Quantification trial using these factors were conducted as follows.

The twenty three parameters composed from real values as well as the Fourier values were supposed to check the significance for FDD, the fault detection and diagnosis. The parameters were reduced by checking the internal correlation and statistical significance. Finally two significant parameters was derived as the most valuable for recognizing difference of patterns between the normal and the faulty ones.

Maximum value of Fourier transform

The difference among the normal and the faulty was observed especially at the $C_k S_l$ zone, as explained in the following and as shown in Figure 3 and 4, as well.

1) The real value along the time axis begins with a larger value and then decreases.
2) The real value along the space axis begins with a smaller value and then increases.
3) The dominant frequency in Fourier analysis gives a characteristic information of temperature profiles. Actually the lower frequency was seen dominant.

Then, the maximum value of the components of the zone defined as described in Equation (3) is selected as one of the parameters.

$$\begin{aligned} & F_m = Max(C_k S_l) \\ & \quad k = N/2, \cdot\cdot, N \qquad l = 1,2, \cdot\cdot, M/2 \end{aligned} \tag{3}$$

Maximum value of frequencies over threshold along the time axis

It is observed that the profiles along the time and space axis varies considerably, if compared with the normal one. It was calculated that the time axis variation was more significant in the FDD. Therefore, the maximum value $\boldsymbol{F_l}$ of the frequencies over a certain threshold value along the time axis was selected as a significant parameter as shown in Equation (4) and (5).

The components $\boldsymbol{Fa_{kl}}$ of the converted matrix ***Fa*** are calculated by filtering with the threshold value ***b*** from the original Fourier matrix [$\boldsymbol{F_{kl}}$] and has the value of either ***k***, ***k***-***N***/2 or 0 depending on its magnitude as shown in Equation (5).

$$F_l = Max(Fa_{kl}) \qquad (4)$$

$$k = 1,2,\cdot\cdot,N \qquad l = 1,2,\cdot\cdot,M$$

$$Fa_{kl} = \begin{bmatrix} \begin{vmatrix} k & k < N/2 \\ k - N/2 & k \geq N/2 \end{vmatrix} & |F_{kl}| > b \\ 0 & |F_{kl}| \leq b \end{bmatrix} \qquad (5)$$

$$k = 1,2,\cdot\cdot,N \qquad l = 1,2,\cdot\cdot,M$$

Data Mapping

The **figure 5** and **Figure 6** show mapping results of all the data obtained from fault simulation on the two dimensional planes, using the parameters described above, for two types of tank. The horizontal axis of them is $\boldsymbol{F_m}$ and the vertical axis is $\boldsymbol{F_l}$. Both of them are normalized using the mean values of all the cases in order to make applicable to any partial load conditions and to any design conditions.

The distributions of the coordinates among normal cases, fault ① cases and fault ② cases are expected to be different each other, which means successful FDD.

DATA GROUPING

How to group the data on the map for each normal and fault case is the next question. The performance of FDD depends upon the quality of grouping method.

The bivariate normal distribution N(μ, σ^2) was supposed for each three group data. The center of the distribution and the probability ellipse are calculated from the enough volume of the sampled data, every group of which has more than seventy data. The Mahalanobis's distance[5][6] $\boldsymbol{D}$ is calculated by the following equations (6) and (7). The ellipses shown in these figures were drawn for one and a half of the standard deviation σ. The most probable answer of grouping for any data is the one that the Mahalanobis' distance is the shortest.

$$D^2 = (x - \mu)^T \sum\nolimits^{-1} (x - \mu) \qquad (6)$$

$$x = \left[x_1 x_2 x_3 \cdot\cdot x_p\right]^T$$

$$\mu = \left[\mu_1 \mu_2 \mu_3 \cdot\cdot \mu_p\right]^T$$

$$\sum = \begin{bmatrix} \sigma_1{}^2 & \sigma_1\sigma_2 \cdots & \sigma_1\sigma_p \\ \sigma_2\sigma_1 & \sigma_2{}^2 \cdots\cdot & \sigma_2\sigma_p \\ \cdot\cdot & \cdot\cdot \quad \cdots & \cdot\cdot \\ \sigma_p\sigma_1 & \sigma_p\sigma_2 \cdots & \sigma_p{}^2 \end{bmatrix} \qquad (7)$$

PERFORMANCE OF FDD

The results of the performance of the fault detection and fault diagnosis obtained from grouping method with Mahalanobis' generalized distance for two types of thermal storage tank are shown in **Table3** for four cases of threshold value $\boldsymbol{b}$ in equation (5), that is, 0.1, 0.3, 0.5 and 1.0.

The fault ② is discriminated from the normal on the mapped area with more than 96% probability of fault detection notwithstanding the $\boldsymbol{b}$ value. In case of the fault ① the probability of detection reduces a little.

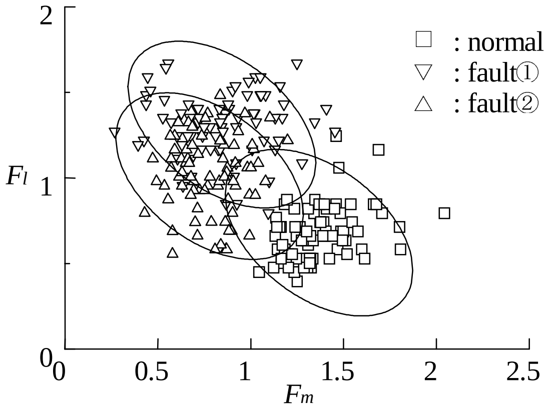


Figure 5 Mapping and grouping normal and faulty data for the multi-connected complete mixing tanks

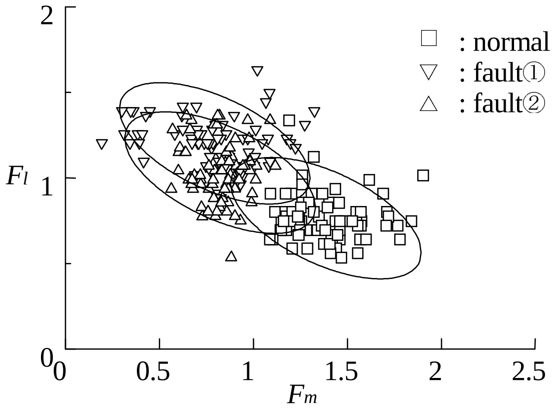


Figure 6 Mapping and grouping normal and faulty data for the single temperature stratified tank

Table 3 Performance of FDD (fault detection / fault diagnosis, or classification)

Threshold	The multi-connected complete mixing tanks		The single temperature stratified tank	
b	Fault①	Fault②	Fault①	Fault②
0.1	0.97 / 0.76	1.00 / 0.76	1.00 / 0.68	1.00 / 0.69
0.3	0.91 / 0.66	0.96 / 0.80	0.93 / 0.63	0.99 / 0.71
0.5	0.88 / 0.68	0.99 / 0.83	0.93 / 0.49	1.00 / 0.64
1.0	0.86 / 0.45	0.99 / 0.91	0.95 / 0.61	1.00 / 0.65

The threshold ***b*** gives little effect on the performance as far as it is below 1.0 and the smaller value looks slightly better, as it can pick up higher frequency components.

The performance of fault diagnosis, or classification, is not so high as the fault detection, because simulation results showed that the temperature profiles of the two kinds of faults sometimes resembles each aother when a certain kind of combination of factors were selected

DISCUSSIONS

In order that the FDD presented in this paper becomes a realistic on-line method, there are several problems to be studied further. One thing is if it is available for the partial load season operation. Authors have already studied it on the simulation basis [4] and reached a conclusion that exactly the same approach using the same parameters for mapping and grouping is effective with almost the same level of performance of FDD.

The second problem is if it is effective even in case of actual installations. Authors are making several trials, too, using actual field data as introduced elsewhere in Japanese [7][8]. The actual operation data has many kind of noises as follows.

1)Measured data are not always correct due to the sensing fault and/or communication noise.

2)It is not clear whether the data come from the normal operation or from the faulty operation.

3)Therefore, it is necessary once to commission the system and obtain the data as normal as possible.

4)The control algorithm and control parameters are not optimized,but rather faultily tuned. In that case, even the normal-like operation generates faulty data.

The item 1) should be solved by maintenance. The item 2), 3) and 4) should be solved by preliminary survey and adjustment of the control system and then gather the normal data beforehand. Usually it should be done just after completion at least for one year.

Other kinds of fault simulation than those described above will give more information which are useful to screen these ‘upstream’ faults, or inoptimality.

CONCLUSION

In order to identify selected thermal storage operation faults, mapping data using appropriate parameters based on Fourier transform on the two dimensional plane and grouping by the statistical principle have proved fairly effective in detecting and localizing faults as follows.

1) Two kinds of control faults, one is malfunction of the three way valve at the primary circuit, **fault①**, and the other is malfunction of the two way valve at AHU and the three way valve for the CDT control in CWV system, **fault** ②, were supposed to take place at 0 o'clock after normal operation.

2)Dynamic fault simulation of the two kinds of reference thermal storage systems was carried out and data for FDD was acquired

3)Variations of the phase, frequency and amplitude was analyzed by way of two dimensional real Fourier transform.

4)In order to quantify the temperature profiles for FDD, two significant parameters composed from Fourier value were selected after checking twenty three parameters with the internal correlation and the statistical significance.

5)The threshold value for the second parameter has not always affected the performance for fault detection and diangnosis as long as it is below 1.0.

6)Data grouping using Mahalanobis' generalized distance on the probability ellipse showed a satisfactory performance of FDD for both types of storage tanks. The parameters available are the same, too.

7)The proposed method is only applicable to those faults which clearly affect the temperature profiles of the thermal storage tank. However, it is believed that most of thermal storage faults affect the temperature profiles, so that the pattern recognition method based on the temperature profiles is considered powerful.

8)Several other problems to be solved remains for the present method to become available for FDD in the real system.

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