OPTIMISATION OF MECHANICAL SYSTEMS IN AN INTEGRATED BUILDING ENERGY ANALYSIS PROGRAM: PART I: CONVENTIONAL CENTRAL PLANT EQUIPMENT

Russell D. Taylor and Curtis O. Pedersen
University of Illinois at Urbana- Champaign
Dept. of Mechanical and Industrial Engineering
1206 W. Green St.
Urbana, IL 61801
USA

ABSTRACT

This is the first of two papers that describe the development of simulation methods for optimally controlled central plant equipment which have been implemented in the IBLAST (Integrated Building Loads Analysis and System Thermodynamics) building energy analysis program. In Part I, conventional central plant equipment is discussed, while Part II covers the simulation of optimally controlled thermal storage systems.

The goal of conventional central plant optimisation is to minimise the operating cost of the plant equipment. A solution was achieved by using a GRG algorithm to minimise the energy consumption of each feasible equipment combination, for every simulation time step, then searching among these combinations for the one which resulted in the least energy cost. Results are presented for several combinations of conventional chilling and heating equipment.

INTRODUCTION

The purpose of this research was to use an integrated energy analysis program to simulate optimal control of a building's central plant equipment. In this paper “control” implies the manipulation of system set points, such as the zone supply air temperature and coil water entering temperature, or the fraction of the load being met by each central plant component to achieve minimum energy cost. In a sense, this is “macroscopic” control compared to “microscopic” control of the system and plant to adjust valves or dampers or to monitor and manipulate duct air pressures to maintain desired flow rates.

The result of the optimisation, in each case, was an hourly schedule of equipment operation that minimised total energy cost taking into account both the hourly variations in energy cost and those assessed according to the maximum plant energy consumption. Scheduling is crucial to plant optimisation since optimisation of components individually does not generally give overall optimum performance (Olson, 1987, 1990, 1993, and 1994).

The building simulation program chosen for this work was IBLAST (Taylor, 1996; Taylor et al, 1990, 1991), a development of the BLAST program which integrates the building zone energy balance with the system and central plant simulation.

When the cost of energy is constant, minimising total energy use is equivalent to minimising energy cost. In contrast, industrial and commercial users of electricity are usually subject to utility rates that vary according to the time-of-use, with increased energy costs during daytime on-peak periods. Additionally, demand charges put a per kilowatt premium on the maximum power consumption experienced during the on-peak hours. This type of rate structure requires a control strategy that directly minimises energy cost. The optimisation problem is complicated because there is no direct relationship between steady state energy consumption and peak demand. Generally, the optimal solution trades increased total energy consumption by operating at less-than-optimal part load efficiency for reductions in peak demand. This requires the forecasting of loads to determine the appropriate schedule of plant control decisions that minimises the peak energy consumption.

METHOD

The objective of this research was achieved by considering the solutions to two separate optimisation problems: minimising the energy consumption of the operating plant equipment at each instant of the simulation, and minimising the total cost of the energy consumed by the system over a daily cycle.

Most large buildings with conventional heating and cooling equipment, e.g. chillers and boilers, normally have several small units of each type of equipment connected in parallel rather than one large unit of equivalent capacity. This allows the operating capacity of the plant to be adjusted to optimise the overall plant part load ratio.

Once the optimal load distribution among the operating plant components is found for an instant of time, the next step is to minimise the energy cost over an interval of time. This requires not only that the equipment combination change to minimise total energy consumption, but also that the peak energy consumption be factored into the problem when demand charges are in effect. Depending on the rate structure, the optimal schedule may require turning the additional equipment on before the on-peak period begins, or operating at a higher than optimal part load ratio rather than turning on more equipment. When demand charges are being considered, the optimisation problem is one of scheduling the available equipment so that it meets the loads as efficiently as possible at each instant of time, while avoiding spikes in energy consumption due to start up transients. Consideration of demand charges in selecting an optimal path through all the possible equipment combinations complicates the problem substantially as discussed by Olson (1987, 1990, 1993, and 1994).

The solution to the optimal load distribution problem required finding the load on each of the operating plant components which minimised the power consumption at the instant the optimisation was performed. The problem first required formulation of the governing equations for the heating and cooling plant. From these equations, several different types of variables were defined according to their function: control variables, that are adjusted by the optimiser to improve plant performance; parameters, constant values obtained from input; and state variables, that describe other aspects of plant performance but are functionally dependent on the controls and the input parameters. Formulation of the state equations was carried out separately for the heating and cooling plant; however, the objective function contained both heating and cooling plant energy consumption information because interactions between the two types of equipment are common. Next, constraint equations were defined to ensure that the state and control variables satisfied the fundamental conservation laws. Finally, the objective function, the quantity being minimised, was formulated. The reader is referred to (Taylor, 1996) for additional details on the formulation of the basic conservation equations which describe the air handling system and central plant operation.

The objective function that was minimised, for each plant combination at every time step, was the total cost of the energy consumed during each simulation time step. That is, the power consumption of each energy type multiplied by the appropriate cost factor as shown in Equation 1:

$$\begin{aligned} J &= C_{elec}\dot{Q}_{elec} + C_{gas}\dot{Q}_{gas} \\ &\quad + C_{\substack{boiler\\fuel}}\dot{Q}_{\substack{boiler\\fuel}} + C_{\substack{diesel\\fuel}}\dot{Q}_{\substack{diesel\\fuel}} \end{aligned} \tag{1}$$

where J is the objective function, C represents the cost factor for each type of energy, and $\dot{Q}$ is the energy consumption rate of each energy type. The constraint equation for the chiller plant was conservation of energy, defined so that the cooling load minus the contribution from each individual chiller equalled zero when the constraint was satisfied as shown in Equation 2:

$$\dot{Q}_{c,load} - \sum_{i=1}^{n_{chillers}} Q_{i,cooling} = 0 \tag{2}$$

Equation 3 is the heating plant's counterpart to Equation 2. Note that waste heat can couple the heating plant to the cooling plant, as can the heat required to operate equipment such as, absorption chillers.

$$\dot{Q}_{h,load} - \sum_{i=1}^{n_{chillers}} Q_{i,waste} - \sum_{i=1}^{n_{boilers}} Q_{i,heating} = 0 \tag{3}$$

The control variables that were varied to minimise the energy cost of the plant were the part load ratios for each plant component $X_{i,plr}$ as defined by:

$$X_{i,plr} = \dot{Q}_i \big/ \dot{Q}_{i,capacity} \tag{4}$$

The part load ratio, $X_{i,plr}$, must also be constrained between maximum and minimum values defined by each piece of equipment.

In order to accomplish the actual optimisation and determine the best equipment operating part load ratios at each time step, a solver was required. A generalised reduced gradient (GRG) routine (GRG User's Guide for UNIX, 1986) was readily available and the problem described above was modified for solution by this method. Implementation of the GRG solver required the development of two additional subroutines: one to control the GRG solver and the other to evaluate the objective function. However, performing the plant equipment optimisation in this manner does not, in general, results in a global minimum. The solver would have to be allowed to modify the other parameters affecting plant operation, such as the plant leaving water temperature, to obtain a true global minimum. The GRG solver was applied at each time step, which could be as short as 1 minute depending on simulation stability requirements. However, the feasible equipment combinations were determined on an hourly basis because of the desire to limit the frequency of changes in operating plant components and the penalties associated with equipment start-up. As a result, only equipment combinations which were feasible for an entire hour were considered when searching for the optimal equipment combination.

It is now appropriate to consider the effect demand charges have on the optimal plant operation schedule. Many utilities impose demand charges on those customers who consume large amounts of energy in addition to subjecting them to variations in the energy cost during a 24 hour cycle. Typically, demand charges are determined by the peak electric power consumption during the utility's billing cycle and is assessed per day of that cycle. For example, Equation 5 is a simple formula for calculating the demand charge is given by:

$$\left(demand\ \ charge\right) = \left(\frac{\$}{day \cdot kW}\right) \times \left(\#\ days\ \ in\ \ cycle\right)\left(peak\ \ kW\right) \tag{5}$$

In practice, the demand charge structure may be considerably more complex and varies between utility companies.

Demand charges penalise peak energy consumption rates, and in order to avoid or at least minimise these charges, consumers must operate their equipment so that the largest peaks in power consumption occur outside the period when peak power consumption is being tracked. Spikes in power consumption are most likely to occur when equipment is turned on because during the start-up sequence operating conditions are far from nominal and additional power is required to accelerate: pumps, fans, motors, etc., up to their design operating speeds. However, start-up spikes are short in duration and typically result in about 20% higher power consumption than steady state operation (Olson, 1987, 1990, 1993, and 1994). Therefore, in order to avoid a large demand charge but maintain sufficient capacity to meet the building loads, equipment may need to be turned on before the on-peak period begins. However, the resulting reduction in demand charges can be more than offset because the total amount of power consumed usually increases.

Clearly, optimising the plant equipment schedule involves a trade-off between increased total energy consumption, because excess plant capacity is maintained to avoid equipment start-up transients, and operating the equipment at an optimal part load ratio but incurring many transients as a result of changes in equipment operation. The incentive to pursue the former strategy increases as the demand charge grows relative to the other operating costs. The latter strategy would be the best option when there is no demand charge since it guarantees the lowest possible power consumption during any and every time interval.

Up to this point the discussion has looked at minimising instantaneous power consumption. Now minimisation of the total energy cost incurred over a specified period of time will be considered. The solution to this type of optimisation problem requires that the sum of all energy costs over the simulation period accounting for different types of fuel, hourly variations in utility rates and demand charges be minimised. This quantity is calculated from Equation 6:

$$C = \sum_{i=1}^{N_{fuel\ types}} c_i Q_{i,tot} + \int_0^T c_{util.elec}(t)\dot{Q}_{elec}(t)dt + c_{demand}\left(\dot{Q}_{elec}^{peak} - \dot{Q}_{elec}^{base}\right) \tag{6}$$

The first term on the right hand side of this equation represents the total cost over the simulation period for energy that has a fixed cost per unit of consumption. That is, the cost of the energy does not vary with time of day. The energy use of fossil fuel powered equipment such as diesel generators and natural gas fired boilers would be included in this term. The second term on the right hand side of Equation 6 is used to calculate the total cost of energy when that cost is time dependent. However, since utility electricity cost normally varies in discreet amounts, not continuously, during the day the second term of Equation 6 can be written as a summation, as in Equation 7:

$$\int_0^T c_{util.elec}(t)\dot{Q}_{elec}(t)dt = \sum_{i=1}^{N_{time\ steps}} c_{util.elec}(t)\dot{Q}_{elec}(t)\Delta t_i \tag{7}$$

The last term on the right hand side of Equation 6 represents utility demand charges. These are assessed per unit power consumption at the peak consumption rate.

The discreet nature of the simulation in terms of both plant equipment choices and the finite time step used to update zone, system, and plant conditions suggests an approach to obtain an optimal equipment schedule which is illustrated in Figures 1, through 4. In these figures, each column represents an hour of the simulation period. Each block in a column represents the state associated with one specific combination of plant equipment. Note that all equipment combinations may not be feasible during a specified hour because of insufficient capacity to meet the heating or cooling loads. For example, in hour 2 combinations 1, 2, and 6 are represented as infeasible. The lines from combinations at one hour to combinations at the next hour represent changes in equipment usage which will, subsequently, be referred to as *transitions*. Each transition has a base power consumption rate associated with it which accounts for the current equipment configuration of the current hour, plus the effects of equipment start-up to obtain a new equipment combination.

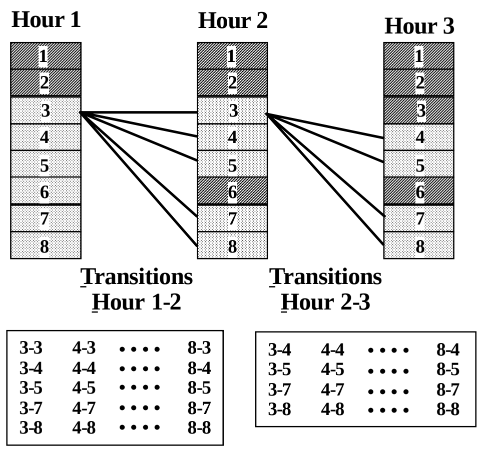

Figure 1: Evaluate All Feasible Equipment Combinations and Transitions

Figure 2 shows the minimum energy consumption path based on the performance of 8 possible equipment combinations. This is determined by selecting, at each hour, the equipment combination with the minimum energy consumption. Repeating this procedure for every hour gives a sequence of equipment operation which results in the minimum total energy consumption. Peak, initial, and final power consumptions are ignored initially. However, these values are needed to determine the demand charge for the minimum energy path and ultimately allow the total energy cost to be computed. It is not possible, *a priori*, to determine whether the peak power consumption results from an equipment transition or the load on the plant equipment. The minimum energy consumption path gives a baseline energy cost and peak power consumption which are compared to the feasible alternative plant equipment schedules, i.e. sequences of plant combinations which differ from the minimum energy plant equipment schedule. In order to determine the actual minimum cost path all the remaining feasible paths must be searched since the peak power consumption may result from an equipment transition between one hour and the next.

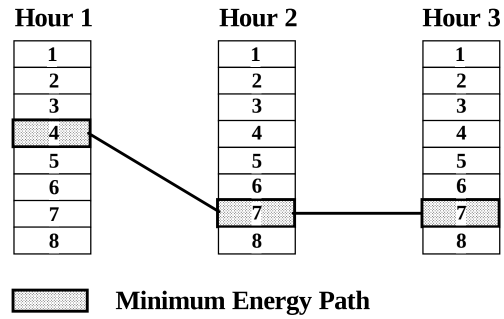

Figure 2: Calculate Minimum Energy Path Based on Hourly Energy Consumption

However, for a path to have a lower total cost than the minimum energy path, it must have a lower peak power consumption because its total energy consumption will, by definition, be higher. As a result, equipment combinations and transitions that have a higher peak power consumption than the minimum energy path may be eliminated, reducing the total number of search paths.

Figure 3 shows how the equipment combination and transition eliminations can substantially reduce the number of search paths required to find the minimum cost path. Finally, the remaining allowable paths are searched to determine the minimum cost path as shown in Figure 4. It should be fairly obvious that unless the number of search paths can be controlled the optimal solution rapidly grows beyond the capabilities of most computers to solve in a reasonable period of time.

While the above description is instructive it does not demonstrate the complexities that occur in plants with many components. Obviously, the more components the more feasible paths that exist. Olson (1987) concluded that an exhaustive search of all the possible paths would be required to find a global minimum because the peak energy consumption of a path is a function of the path itself. This conclusion led Olson to use heuristic methods to find the optimal path because the alternative, an exhaustive search, was impractical. However, the resulting solution was never guaranteed to be a global minimum. With one caveat, the method described above represents an alternative to Olson's heuristic approach that does result in a global minimum energy cost without requiring an exhaustive search. The caveat is that while the peak power consumption experienced during equipment start-up is factored into the determination of the optimal path, the additional energy consumed as a consequence is neglected. Assuming that chiller power requirements are nominally 50% higher than steady state for a period of 0.1 hours then the net increase in energy consumed is 5% over the value for steady state chiller operation for a single chiller. If other equipment is already operating when the chiller is turned on then this effect is diluted considerably.

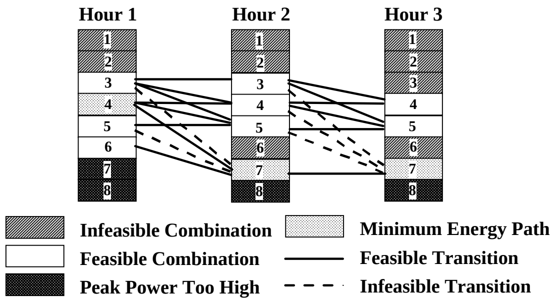

Figure 3: Determine Feasible Paths to Search by Eliminating Combinations and Transitions

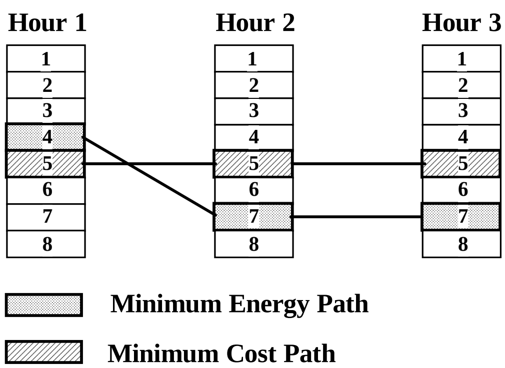

Figure 4: Determine Minimum Cost Path by Comparing Feasible Paths to Minimum Energy Path

Equation 8 shows that an arbitrary initial path has costs derived from both total energy consumption and peak consumption. A good choice for the initial path is the minimum energy path because it can be determined easily. The total cost associated with this path is:

$$C_{ref} = C_1 Q_{tot}^{\min} + C_2 \dot{Q}_{peak}^{\min} \qquad (8)$$

If the demand charge C_2 is zero then this is the least cost path. However, if C_2 is non zero then this path may or may not also be the minimum cost path. The total cost of any other arbitrary but feasible path is given by Equation 9:

$$C_i = C_1 Q_{tot}^{i} + C_2 \dot{Q}_{peak}^{i} \qquad (9)$$

But, for any other path, the total energy consumed must be greater than the energy consumed by the least cost path. This is given by Equation 10:

$$Q_{tot}^{i} > Q_{tot}^{\min} \qquad (10)$$

Therefore, for the total cost of path C_i to be less than C_{ref}, Equation 11 must hold:

$$\dot{Q}_{peak}^{i} < \dot{Q}_{peak}^{\min} \qquad (11)$$

This means that any path segment, or transition, between equipment configurations having a higher peak consumption than the reference path can be eliminated from further consideration. The global minimum may thus be found by searching on a much reduced subset of the feasible paths.

RESULTS

In order to demonstrate the feasibility of the optimisation scheme described above, a central plant consisting of two 17kW electric powered chillers, one 44kW electric powered chiller, one 23kW diesel powered chiller, and one 5kW electric boiler to supply domestic hot water was specified to serve a two zone building with a VAV system. The influences of the relative cost of each type of energy and the demand charge on the least cost equipment schedule were of most interest.

In the first set of examples, the demand charge was 200/kW and the on-peak rate multiplier was 2. Diesel cost was given values of 1.5/kWh, 3/kWh, 5/kWh, and 10/kWh to demonstrate how the minimum cost schedule would change with this parameter. Figure 5 shows the minimum energy equipment schedule when the cost of diesel fuel is 1.5/kWh. This schedule does not utilise the diesel chiller at all and the electric boiler is used to supply domestic hot water. Figure 6, the minimum cost schedule, provides a significant contrast since the diesel chiller operates during hours 13 through 17 and the boiler does not run because waste heat from the diesel chiller meets the domestic hot water load. However, during hour 12 the diesel chiller produced insufficient waste heat and the boiler operated at a minimal part load fraction. During hour 14, a transition causes all the chillers to operate and much more capacity is available than for the minimum energy schedule. In this situation, the peak electric consumption is a steady state peak and is not due to an equipment start-up transient. Further analysis indicated that turning on the diesel chiller shifted the peak electric consumption to hour 12, in the minimum cost schedule, from hour 14, in the minimum energy schedule. In addition, a significant reduction in peak electric consumption occurred because the diesel chiller allowed the electric powered chillers to operate at lower part load fractions.

Figure 7 shows the hour-by-hour difference in total energy consumption between the minimum energy and minimum cost schedules represented in Figures 5 and 6. The least cost schedule is entirely the result of the demand charge, otherwise the total cost of the energy consumed by the minimum energy path is lower than the total energy cost of the minimum cost path.

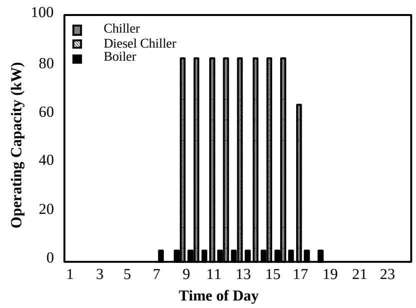

Figure 5: Minimum "Energy" Plant Operating Schedule for a Combination of Electric and Diesel Chillers and an Electric Boiler (Diesel Cost = 1.5/kWh)

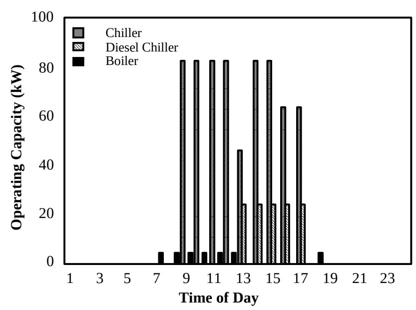

Figure 6: Minimum Cost Plant Operating Schedule for a Combination of Electric and Diesel Chillers and an Electric Boiler (Diesel Cost = 1.5/kWh)

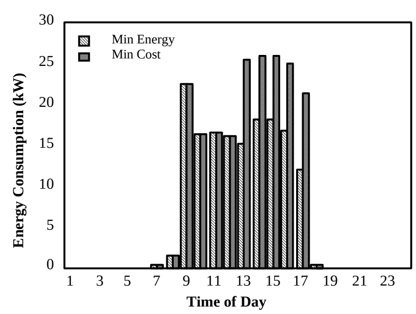

Figure 7: Difference in Actual Energy Consumption Between Minimum Energy and Minimum Cost Schedules (Diesel Cost = 1.5/kWh)

Figure 8 shows how increasing the cost of diesel fuel affects the difference in total cost between the minimum cost and minimum energy schedules. When diesel costs less than about 5.5/kWh the minimum cost path uses the diesel chiller during the on-peak hours, as shown in Figure 6. However, when the cost of diesel fuel is greater than this value, the total cost, including demand charge, for using all electric chillers becomes less than for using a combination of diesel and electric chillers, even with the reduction in peak electricity demand.

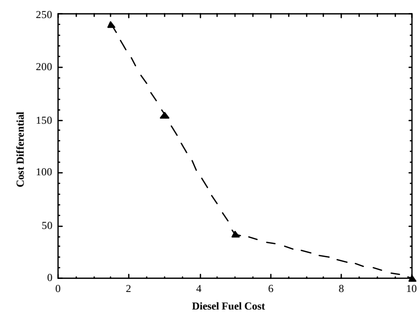

Figure 8: Total Cost Differential Between Minimum "Energy" and Minimum Cost Schedules as Diesel Fuel Cost is Increased

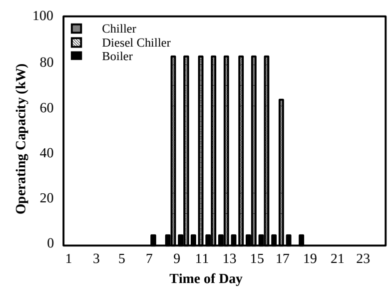

Figure 9: Minimum "Energy" Plant Operating Schedule for a Combination of Electric and Diesel Chillers and a Fuel Fired Boiler (Diesel Cost = 2.0/kWh)

A second example was generated by replacing the electric boiler with a fuel fired boiler of the same capacity. In addition, a diesel fuel cost of 2.0/kWh and a boiler fuel cost of 0.5/kWh were specified. Figure 9 shows that the minimum energy equipment schedule for this case is identical to the plant schedule shown in Figure 5. However, the minimum cost plant schedule, shown in Figure 10, is significantly different from the corresponding figure of the previous example, Figure 6.

The minimum cost path in this example still utilised the fuel boiler for hour 12 but the remaining electric chiller was not turned on at the end of hour 13, as in the previous case. In the previous example, the peak electric consumption occurred at hour 12 for minimum cost path and the electric boiler was turned on, contributing to the peak electricity load. In this example, the boiler is fuel fired so that, although the peak electric consumption also occurs at hour 12, it is a lower peak. Turning on the chiller at hour 14, as in the previous example, would increase peak electric consumption and move the time at which it occurs to hour 14. Not turning on the additional chiller and operating all the equipment at overall higher part load fractions to meet the cooling load avoided this peak in consumption.

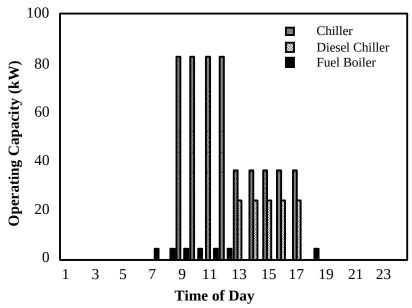

Figure 10: Minimum Cost Plant Operating Schedule for a Combination of Electric and Diesel Chillers and a Fuel Fired Boiler (Diesel Cost = 2.0/kWh)

The equipment from the previous example was also used to evaluate the sensitivity of the minimum energy and minimum cost equipment schedules to variations in demand charge and on-peak electricity rate multiplier. The demand charge was varied from 200/kW to 1/kW for a fixed value of the on-peak rate multiplier and four on-peak electric rate multiplier values were used: 2, 4, 6, and 10. The results are given in Figure 11, showing the additional cost of the minimum energy schedule compared to the minimum cost plant operating schedule.

As the demand charge was increased, in all cases except for an on-peak rate multiplier of 10, the difference in cost between the two schedules increased. The minimum energy and minimum cost paths also converged when the demand charge was reduced to less than 10/kW. These results matched those obtained in previous cases except when the on-peak electric rate multiplier was 10. An example of this case is plotted in Figure 12 for a demand charge of 200/kW, showing the on-peak use of diesel chillers because, per unit cooling, electricity was more expensive than diesel fuel. Another effect was the raising of the diesel chiller part load ratio at hour 12 so that the boiler was not required to supply any of the domestic hot water load.

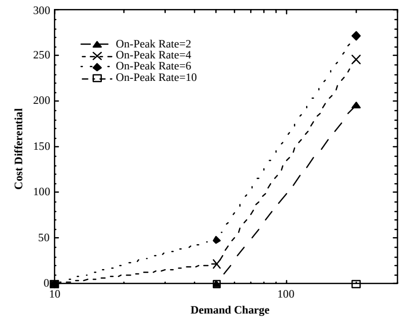

Figure 11: Effect of Demand Charge and On-Peak Electric Rate On the Difference Between Minimum Energy and Minimum Cost Schedules (Diesel Cost = 2.0/kWh)

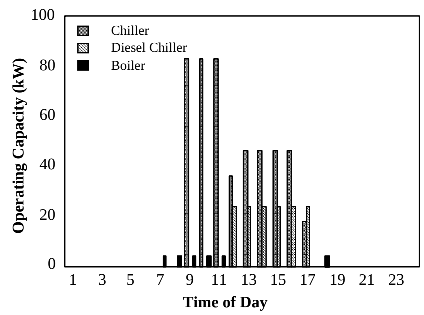

Figure 12: Optimal Plant Equipment Schedule when On-Peak Electricity = 10/kWh, Demand Charge = 200/kW and Diesel = 2.0/kWh

CONCLUSIONS

In order to determine optimal plant equipment schedules, the feasible optimal equipment combinations at each time step had to be evaluated. The optimisation was based on selecting the part load ratio at which each piece of available equipment was to be operated. However, when more than one equipment type was used, and especially when the different types were powered by different energy sources, the optimal solution was strongly dependent on the relative difference between the energy rates and, when applicable, their variations during a one day cycle.

The optimal equipment schedules described in this research were determined by piecing together, for each hour of the simulation, a series of equipment combinations whose energy cost had been minimised by selecting the appropriate part load ratios for each piece of equipment. Normally, in order to find the lowest cost schedule, a search would have to be conducted on all the feasible paths resulting in the total number of possible paths increasing until required computation time and data storage became prohibitively large. An alternative to a global search of all possible paths was used in solving this problem. This alternative recognised that only a limited subset of the possible paths need be considered as candidates for a true global minimum. This subset was made up of those paths that had lower peak energy consumption than the reference minimum energy. In cases where the number of possible equipment combinations was large, the reduction in the total number of paths calculated was typically several orders of magnitude. Furthermore, the method finds a true global minimum energy cost under the assumption that the cost of the additional energy consumed during an equipment start-up spike is negligible.

The optimal scheduling method developed in this paper provided results that follow expected trends. For example, as the utility demand charges increase, the simulation model predicts that fewer changes in equipment operating capacity should occur. However, the results do not lead to obvious generalisations or rules-of-thumb. The wide selection in equipment types and the variations in energy costs imply that a simulation of the proposed building and its equipment is necessary to accurately determine the best way to operate equipment.

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NOMENCLATURE

Variables

c_{demand} - demand charge cost factor

$c_{util,elec}$ - utility electric cost factor

t - time

C - Cost coefficient

J - Value of the objective function

Q_{tot} - Total energy consumption of a path

$\dot{Q}$ - Power consumption

$\dot{Q}_{peak}$ - Peak power consumption along a path

$\dot{Q}_{elec}^{base}$ - Power consumption threshold for calculating demand charge

$\dot{Q}_{elec}^{peak}$ - Peak power consumption for calculating demand charge

T - temperature

X - Equipment part load ratio

Superscripts

min - value associated with the minimum energy path

Subscripts

peak - maximum associated with a path

ref - associated with a reference path