EVALUATION OF THE FINITE CONTROL VOLUME METHOD IN SIMULATING THERMAL FIRE RESISTANCE OF BUILDING ELEMENTS

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ABSTRACT

Application of the finite control volume method on simulating thermal fire resistance of building materials and elements was evaluated. Example was taken on studying the thermal responses of a concrete column under fire. By neglecting moisture transfer, the thermal conduction equation in concrete was solved numerically to get the temperature distribution. Results were compared with those calculated from other finite difference schemes including the forward difference, backward difference and central difference schemes. Methods on handling the transient term including the explicit method, the implicit method, the Crank-Nicolson method and the Alternating-Direction-Implicit scheme were considered. Numerical problems encountered in using those schemes were listed and quantified. Steady state results were compared with analytical solution. Recommendation was then made on selecting the appropriate finite difference scheme for simulating the thermal fire resistance of building element for design purposes.

(1) INTRODUCTION

"Fire resistance" is a term associated (e.g. BS476-1987, Malhotra 1982) with the ability of a building element to perform its usual function when exposed to fire. Fire resistance periods are specified for different building elements to denote how long they can stand without loss in its function as loadbearing or prevention of fire spreading. Studying the thermal fire resistance of building structures and elements is essential and standard tests such as the BS476 part 20 are performed for evaluating the fire resistance of structures and building components. There, a furnace (either by oil or by gas fuel) is built to heat up the structure for testing its stability, integrity and insulation.

Stability is the ability for the element to carry the load without collapsing. Integrity is the ability to prevent the formation of cracks or perforations so that smoke and flame cannot spread from one side to the other side of the element. Insulation is the ability to prevent the passage of heat through the element.

Carrying out a fire resistance test is very expensive and time-consuming. In addition to the testing time, a waiting period of up to 1 week might be required for cooling the furnace. There are no laboratories in Hong Kong equipped with such a resistance furnace, and computer simulation (Wade and Kvokosky 1982, Calhoun 1983, Hakseuer 1985, Sterner and Wickstron 1990, Fields and Fields 1991, Chow and Chan 1996) will be useful for assessing the fire resistance of structures and new building products.

Analysis of the fire resistances for the non-combustible materials can be described as a thermal conduction problem. Numerical methods are employed to solve the thermal equations concerned (e.g. Chow and Chan 1996). Once the failure criteria are specified, the model can be applied to predict the thermal fire resistance. However, many numerical methods are available in the literature and no comparison among them for fire resistance simulations appeared in the literature.

In this paper, the use of the finite control volume method (Patankar 1981, Versteeg and Malalasekera 1995) to study the thermal fire resistance of materials was evaluated. This is the first stage of the numerical studies on the fire behaviour of building materials. The objective is to investigate how far the finite control volume method can go and so only thermal conduction is involved at the moment. A concrete column was taken as the example. Results predicted by the finite control volume method are compared with other finite difference methods. Comparisons with analytical solutions under steady-state conditions are also made. Later stages of study will be on including other effects such as moisture transfer for concrete (e.g. Harmathy 1971).

(2) THE PHYSICAL PROBLEM

In determining the fire resistane of a building element, one side of the element is exposed to a fire furnace with a standard temperature time curve. Heat is then transferred in the material by conduction if combustion or phase transition processes are neglected. The equation [in Cartesian co-ordinate system (x, y, z)] for the temperature T of the material at time t after starting the fire can be expressed in terms of the density ρ, specific heat capacity c and thermal conductivity k (Carslaw and Jaeger 1959):

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}$$
$$=\rho c\frac{\partial T}{\partial t} \qquad \text{... (1)}$$

Defining the thermal diffusivity α as $\frac{k}{\rho c}$ and transforming equation (1) into cylindrical co-ordinate system (r, θ, z) for constant thermal conductivity k:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial\ T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{... (2)}$$

A concrete column as shown in Fig. 1 was considered in this paper.

The density of concrete ρ is taken as 2300 kgm^{-3}, the thermal conductivity k is 1279 $kWm^{-1}K^{-1}$, specific heat capacity C is 1130 $Jkg^{-1}K^{-1}$ and so the thermal diffusivity α is 0.4921 m^2s^{-1}.

Suppose the upper surface of the column is exposed to the fire furnace as in Fig. 1, in this way, the effect of fire occurring at a upper level can be investigated. This is an axisymmetric problem and there is no heat generation term inside the material, equation (2) becomes :

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{... (3)}$$

(3) FINITE CONTROL VOLUME METHOD

For the finite control volume (CV) method (Patankar 1981), a control volume about a point P as in Fig 2 is considered. Integrating both sides of equation (3) over the control volume:

$$\int_s^n \int_w^e \int_t^{t+\Delta t} \frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right) dzdrdt +$$
$$\int_s^n \int_w^e \int_t^{t+\Delta t} \frac{1}{r}\frac{\partial T}{\partial r} dzdrdt +$$
$$\int_s^n \int_w^e \int_t^{t+\Delta t} \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) dzdrdt$$
$$= \frac{1}{\alpha}\int_s^n \int_w^e \int_t^{t+\Delta t} \left(\frac{\partial T}{\partial t}\right) dzdrdt \quad \text{... (4)}$$

Rearranging and simplifying this equation, the temperature $T_{i,j}^{k+1}$ at the point P (at position i,j of the grid system with i along r and j along z.) and time t (at interval k+1) is expressed in terms of the values $T_{i+1,j}^{k+1}$, $T_{i-1,j}^{k+1}$, $T_{i,j+1}^{k+1}$ and $T_{k,j-1}^{k+1}$ in the neighbourhood points E, W, N, S; and the value at the previous time (at interval k) $T_{i,j}^{k}$ as :

$$a_{i,j}^{k+1}T_{i,j}^{k+1} = a_{i,j}^{k}T_{i,j}^{k} + a_{i+1,j}^{k+1}T_{i+1,j}^{k+1} + a_{i-1,j}^{k+1}T_{i-1,j}^{k+1} + a_{i,j+1}^{k+1}T_{i,j+1}^{k+1} + a_{i,j-1}^{k+1}T_{i,j-1}^{k+1} \quad \text{... (5)}$$

where

$$a_{i+1,j}^{k+1} = \frac{(\Delta z)}{(\Delta r)_e} + \frac{(\Delta z)}{2r_{i,j}}$$
$$a_{i-1,j}^{k+1} = \frac{(\Delta z)}{(\Delta r)_w} - \frac{(\Delta z)}{2r_{i,j}}$$
$$a_{i,j+1}^{k+1} = \frac{(\Delta r)}{(\Delta z)_n}$$
$$a_{i,j-1}^{k+1} = \frac{(\Delta r)}{(\Delta z)_s} \quad \text{... (6)}$$

$$a_{i,j}^{k} = \frac{(\Delta r)(\Delta z)}{(\Delta t)\alpha} \quad \text{... (7)}$$

$$a_{i,j}^{k+1} = a_{i+1,j}^{k+1} + a_{i-1,j}^{k+1} + a_{i,j+1}^{k+1} + a_{i,j-1}^{k+1} + a_{i,j}^{k} \quad \text{... (8)}$$

The boundary conditions for temperature at i = 0 and j = 0 for N nodes $T_{0,j}$, $T_{N,j}$, $T_{i,0}$ and $T_{i,N}$ (all in ^{o}C) are:

$$T_{0,j} = T_{N,j} = 20,$$

$$T_{i,0} = 20,$$

$$T_{i,N} = [345\log(8t+1)+20]$$

The Gauss-Seidel method was used to solve the system of equations (5) iteratively. Computation would be stopped when the difference in subsequent iteration was less than 10^{-6} or over 1000 iterations were performed.

(4) SPATIAL PART

Other finite difference schemes were also used to solve the spatial part of equation (3). Those are the central difference scheme, forward difference scheme and the backward difference scheme. The central difference (CD) scheme is (neglecting the time k in this section for spatial analysis) :

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta r)^2} + \frac{1}{r_{ij}}\frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta r)} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta z)^2} = 0 \quad \text{... (9)}$$

The forward difference (FD) scheme is:

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta r)^2} + \frac{1}{r_{ij}}\frac{T_{i+1,j} - T_{i,j}}{(\Delta r)} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta z)^2} = 0 \quad \text{... (10)}$$

The backward difference (BD) scheme is:

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{(\Delta r)^2}+\frac{1}{r_{ij}}\frac{T_{i,j}-T_{i-1,j}}{(\Delta r)}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{(\Delta z)^2}=0 \quad \text{... (11)}$$

For all cases, the temperature $T_{i,j}$ at the point P at (i,j) can be obtained by solving the linear system using Gauss-Seidel iteration method.

$$T_{i,j}b_{i,j}=T_{i+1,j}a_{i,j}+T_{i-1,j}c_{i,j}+T_{i,j+1}(\Delta r)^2+T_{i,j-1}(\Delta r)^2 \quad \text{... (12)}$$

where $a_{i,j}$, $b_{i,j}$ and $c_{i,j}$ for the three schemes are shown in Table 1.

(5) DISCRETIZATION FOR TRANSIENT PROBLEM

Four schemes were considered for handling the transient part of equation (3). Those are the Crank-Nicolson (CN) scheme, the implicit scheme (IS), the explicit scheme (ES) and the Alternating-Direction-Implicit (ADI) scheme. For central difference scheme in handling the spatial part, equation (3) is discretized as :

$$f\left(\frac{T^{k+1}_{i+1,j}-2T^{k+1}_{i,j}+T^{k+1}_{i-1,j}}{(\Delta r)^2}+\frac{1}{r_{ij}}\frac{T^{k+1}_{i+1,j}-T^{k+1}_{i-1,j}}{2(\Delta r)}+\frac{T^{k+1}_{i,j+1}-2T^{k+1}_{i,j}+T^{k+1}_{i,j-1}}{(\Delta z)^2}\right)+(1-f)\left(\frac{T^{k}_{i+1,j}-2T^{k}_{i,j}+T^{k}_{i-1,j}}{(\Delta r)^2}+\frac{1}{r_{ij}}\frac{T^{k}_{i+1,j}-T^{k}_{i-1,j}}{2(\Delta r)}+\frac{T^{k}_{i,j+1}-2T^{k}_{i,j}+T^{k}_{i,j-1}}{(\Delta z)^2}\right)=\frac{1}{\alpha}\frac{T^{k+1}_{i,j}-T^{k}_{i,j}}{\Delta t} \quad \text{... (13)}$$

The factor f was taken as 1/2 for the Crank-Nicolson scheme, 1 for the implicit scheme and 0 for the explicit scheme. However, no converged solution for explicit scheme would be obtained if the time step Δt does not satisfy:

$$\frac{\alpha\,\Delta t}{(\Delta r)^2+(\Delta z)^2}\leq\frac{1}{8} \quad \text{... (14)}$$

For the Alternating-Direction-Implicit scheme, the solution is obtained by alternating direction iterations. For r-direction,

$$\left(\frac{T^{k+1}_{i+1,j}-2T^{k+1}_{i,j}+T^{k+1}_{i-1,j}}{(\Delta r)^2}+\frac{1}{r_{ij}}\frac{T^{k+1}_{i+1,j}-T^{k+1}_{i-1,j}}{2(\Delta r)}+\frac{T^{k}_{i,j+1}-2T^{k}_{i,j}+T^{k}_{i,j-1}}{(\Delta z)^2}\right)=\frac{1}{\alpha}\frac{T^{k+1}_{i,j}-T^{k}_{i,j}}{\Delta t} \quad \text{... (15)}$$

For z-direction,

$$\left(\frac{T^{k+1}_{i+1,j}-2T^{k+1}_{i,j}+T^{k+1}_{i-1,j}}{(\Delta r)^2}+\frac{1}{r_{ij}}\frac{T^{k+1}_{i+1,j}-T^{k+1}_{i-1,j}}{2(\Delta r)}+\frac{T^{k+2}_{i,j+1}-2T^{k+2}_{i,j}+T^{k+2}_{i,j-1}}{(\Delta z)^2}\right)=\frac{1}{\alpha}\frac{T^{k+2}_{i,j}-T^{k}_{i,j}}{\Delta t} \quad \text{... (16)}$$

(6) RESULTS

The temperature distribution in the concrete column were predicted by dividing the concrete column into 121 parts with the grid system shown in Fig 1. A computer package was developed using Visual C++ with program listing available either from the authors. A large volume of data was found and it is difficult to present all the output. A monitoring point M shown in Fig 1 was considered and the transient variation of temperature at this point predicted by the control volume CV scheme is shown in Fig 3.

Results predicted by the central difference CD method with the four transient schemes (CN, IS, ES and ADI) are also shown in Fig 3. Results obtained were similar to those predicted by the CV scheme.

For comparing with the other finite difference methods (FD and BD in addition to the CD) for handling the spatial part of equation (3), the upper part of the boundary was set at 918 ^{o}C, corresponding to 50 s in the BS476 fire curve. Converged results were not obtained in using the BD scheme. Results on the temperature contours for CV, CD and FD are shown in Fig 4. The temperature distribution patterns obtained from the CD and FD schemes are similar to that by CV.

Exact solution is available for a cylinder of radius a and length L with one face kept at a prescribed temperature f(r) and the other surface at zero. The temperature T at r and z is :

$$T = \frac{2}{a^2} \sum_{n=1}^{\infty} \frac{J_0(r\alpha_n)\sinh\left(L-z\right)\alpha_n}{J_1^2\left(a\alpha_n\right)\sinh(L\alpha_n)} \int_0^a rf(r)J_0\left(r\alpha_n\right)dr \quad \text{... (17)}$$

where the α_n are the positive roots of $J_0\left(a\alpha_n\right) = 0$

The function f(r) is taken to be at constant temperature 918^{o}C and the same monitoring point M was considered. Since f(r) is set to be a constant (918°C), and

$$\int x^n J_{n-1}(x)dx = x^n J_n(x) \quad \text{... (18)}$$

Giving the temperature evaluated from the analytical expression T_{ANA} :

$$T_{ANA} = \frac{2f(r)}{a} \sum_{n=1}^{\infty} \frac{J_0(r\alpha_n)\sinh(L-z)\alpha_n}{J_1(a\alpha_n)\sinh(L\alpha_n)\cdot\alpha_n} \quad \text{... (19)}$$

Values of T_{ANA} are 50.3°C, 50.2°C, 50.2°C and 50.2°C by keeping the first term (n=1), second term (n=2), third term (n=3) and fourth term (n=4) in the series. Therefore, 50.2°C is a good estimation of T_{ANA}. The value is a good reference to check the results predicted by different numerical methods.

Results predicted by the finite difference schemes T_{FD} and the analytical solution T_{ANA} are shown in Table 2. The percentage derivation PD between the two approaches are calculated by:

$$PD = \frac{T_{FD} - T_{ANA}}{T_{ANA}} \text{x}100\% \quad \text{... (20)}$$

Values are also compared with these values T_{CD} and T_{CV} by CD and CV schemes. It can be seen that the control volume gave the best estimation. The temperatures were expressed in ^{o}C which is used commonly in the local industry. It is understood that results on PD will be very different when the temperatures are in K.

Table 2: Comparison of different schemes

	T_{ANA}	T_{CD}	T_{CV}	T_{FD}
Temp. /°C	50.2	40.6	48.1	37.8
PD/%	0	19	4	25

(7) CONCLUSIONS

Applying the finite control volume method to study the thermal fire resistance for a concrete column is illustrated. The method takes the advantage of using coarser grids. Results predicted are more reasonable as heat transferred to the interior parts with a practical rate.

Experimental studies have to be performed for validating the results.

This is only the first step in evaluating the approach and further reports will be presented. Detailed investigations should include the moisture transfer (Harmathy 1971, Huang and Ahmed 1989) for the case of concrete. But before doing so, the finite difference scheme to be selected has to be evaluated. It is observed from this study that the finite control volume method is a good scheme for simulating fire resistance of concrete ; and possibly for timber with combustion.

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Table 1: Summary of coefficients in the three schemes

Coefficient	a_{ij}	b_{ij}	c_{ij}
Central difference (CD)	$(\Delta z)^2+\frac{1}{r_{ij}}\left(\frac{1}{2}\right)(\Delta r)(\Delta z)^2$	$2(\Delta z)^2+2(\Delta r)^2$	$(\Delta z)^2-\frac{1}{r_{ij}}\left(\frac{1}{2}\right)(\Delta r)(\Delta z)^2$
Forward difference (FD)	$(\Delta z)^2+\frac{1}{r_{ij}}(\Delta r)(\Delta z)^2$	$2(\Delta z)^2+\frac{1}{r_{ij}}(\Delta r)(\Delta z)^2+2(\Delta r)^2$	$(\Delta z)^2$
Backward difference (BD)	$(\Delta z)^2$	$2(\Delta z)^2-\frac{1}{r_{ij}}(\Delta r)(\Delta z)^2+2(\Delta r)^2$	$(\Delta z)^2-\frac{1}{r_{ij}}(\Delta r)(\Delta z)^2$

NOMENCLATURE

a_i	coefficient in finite difference equation
T	temperature (in °C)
$\dot{q}$	heat generated rate per unit volume
c	specific heat kJ/kg°C
t	time
P, N, S, E, W	positions in a control volume
$\frac{\partial}{\partial t}$	partial derivatives with respect to time
r, θ, z	cylindrical co-ordinates
$\frac{\partial T}{\partial r}, \frac{\partial T}{\partial \theta}, \frac{\partial T}{\partial z}$	partial derivatives with respect to r, θ, z
x, y, z	Cartesian co-ordinates
$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$	partial derivatives with respect to x, y, z
(Δr) (Δz) $(\Delta z)_s$ $(\Delta z)_n$ $(\Delta r)_w$ $(\Delta r)_e$	elements of control volume
$r_{i,j}$, $z_{i,j}$	co-ordinates in the discretization equation

GREEK SYMBOLS

α	thermal diffusivity (in m^2s^{-1})
ρ	density (in kgm^{-3})

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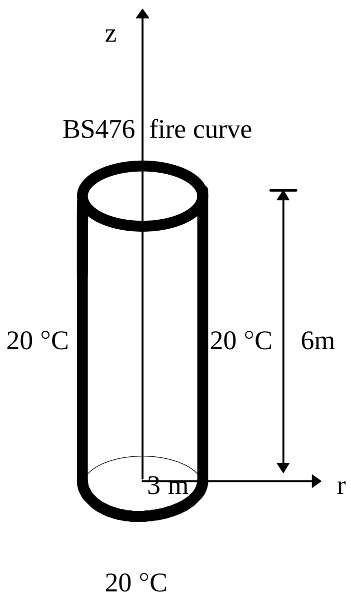
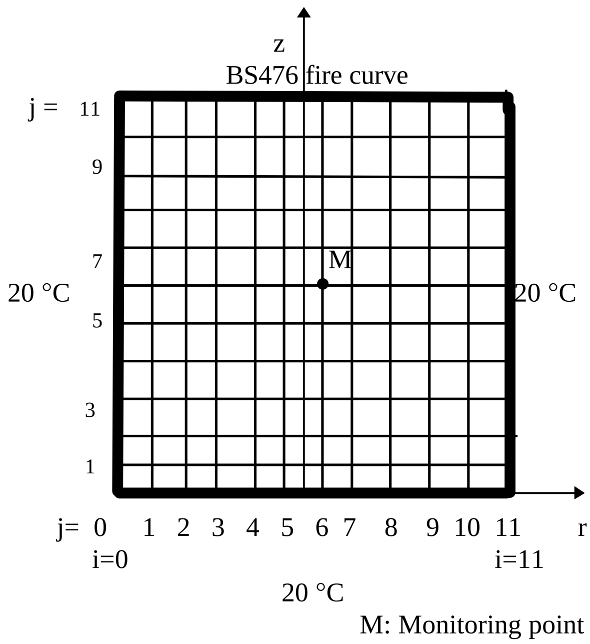


(a) Geometry



(b) Grid system

Fig. 1: The Concrete Column

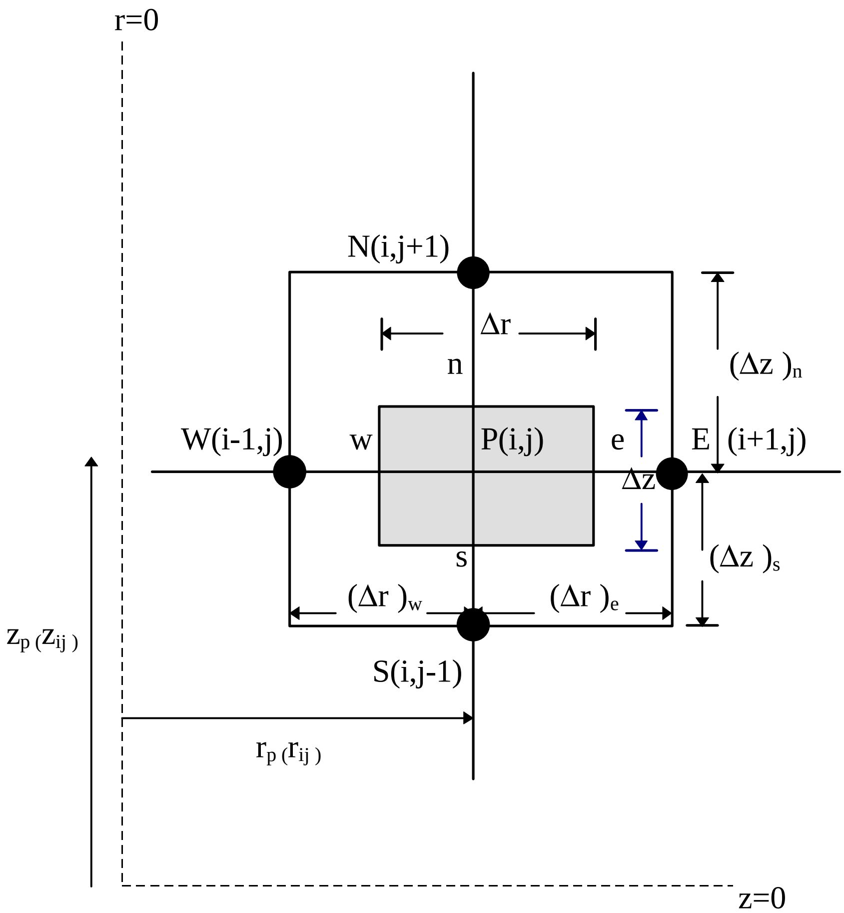


Fig. 2 : Control Volume

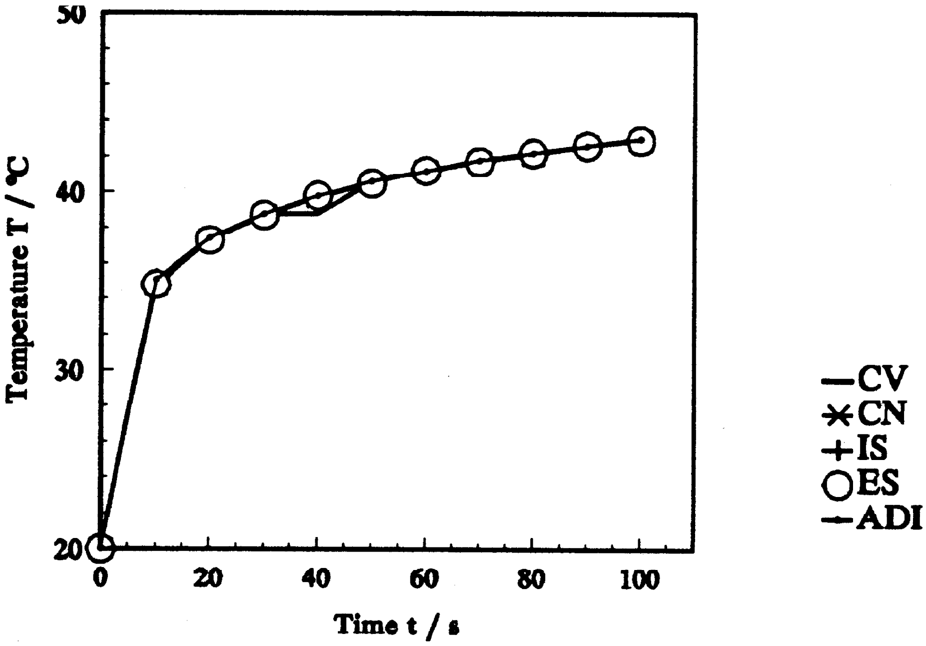
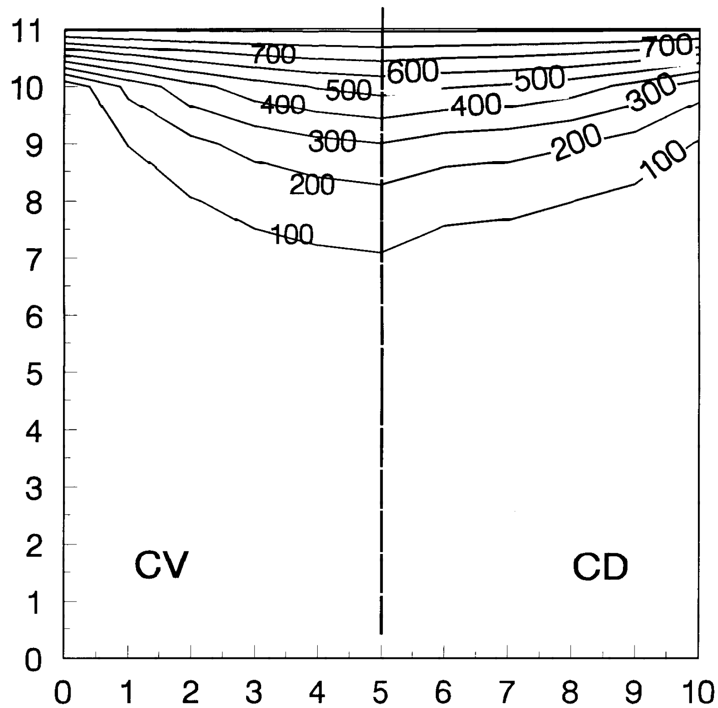


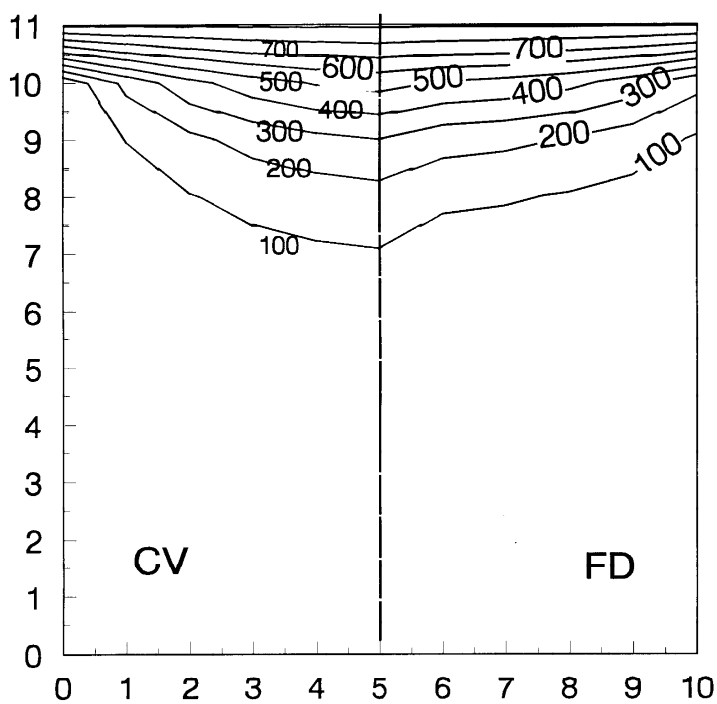
Fig 3 : Transient temperature at point M

BS 476 fire curve at 50s



(a) Central Difference

BS 476 fire curve at 50s



(b) Forward Difference

Fig. 4: Temperature contours from different schemes