PARAMETER IDENTIFICATION TO ANALYSE HEAT INSULATION MEASUREMENTS

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ABSTRACT

In this paper a new procedure of determining the thermal resistance and the thermal capacity of multilayer walls is described. Its feature is the simulation of the thermal behaviour of the wall with subsequent parameter identification. The possibilities and limits of the procedure are shown and filustrated with results from an one- and a two-layer wall. A comparison of the described method with the wellImown averaging method shows the advantages in using dynamic analysis.

INTRODUCTION

The thermal properties of building materials can be exactly determined in a laboratory. However, for an evaluation of the energetical behaviour of a building the changes in these properties caused by the conditions of use must be taken into account. The real values are obtainable only by field measurements. Because of the difficulties in the realization and interpretation of such measurements a lot of corresponding methods have been developed in the past. In most of them the considered wall is described by a thermal network or by an abstract set of mathematical equations (black box models). In these cases the influence of the model error on the accuracy of the results is difficult to estimate.
In contrast to this, the method described in this paper uses a purely physical model (white box model). Only the one-dimensional heat conduction equation for multi-layer systems is postulated.

DESCRIPTION OF THE METHOD

The method only requires the measurements of the surface temperatures at the interior ($T_i(t)$) and the exterior side of the wall ($T_e(t)$) as well as the measurement of the heat flow density at the interior side ($q_m(t)$) during a period of some days. The thermal conductivities λ_j, the thermal diffusivities α_j and the thicknesses d_j of each individual layer are taken as free parameters, whereas the total thickness of the wall is taken as a fixed (and known) value. Thus, in case of a one-layer wall there are only two parameters (λ and α) while in case of a two-layer wall there are five parameters (λ and α of each layer and the thickness of one of the layers). As a result of the calculations values are obtained for this parameters, from which further values are calculable, e.g. the total thermal resistance of the wall

$$R = \sum_{j=1}^{K} \frac{d_j}{\lambda_j} \tag{1}$$

(K denotes the number of layers) or the total thermal capacity per area

$$C = \sum_{j=1}^{K} \frac{\lambda_j \cdot d_j}{\alpha_j} \tag{2}$$

With the measured temperatures $T_i(t)$ and $T_e(t)$ and initial values of the parameters the heat conduction equation

$$\frac{\partial T}{\partial t} = \alpha(x) \cdot \frac{\partial^2 T}{\partial x^2} \tag{3}$$

can be solved numerically (finite difference procedure) to obtain the time dependent temperature field $T(x,t)$ over the cross-section of the wall. $\alpha(x)$ denotes the thermal diffusivity of the actual layer. By inserting the first derivation of the temperature field $\partial T/\partial x$ into the Fourier equation

$$q(x,t) = -\lambda \cdot \frac{\partial T}{\partial x} \tag{4}$$

the heat flow density $q_c(t) = q(0,t)$ at the interior side of the wall can be calculated (λ denotes the thermal conductivity at the interior side). This calculated heat flow density $q_c(t)$ must be compared to the measured one $q_m(t)$. Now the free parameters can be varied as long as the best possible conformity between both heat flow densities is reached. As a criterion for this conformity the function Chi^2 is taken, that means the sum of the squares of all deviations:

$$Chi^2(P) = \sum_{j=1}^{N} (q_{m,j} - q_{c,j}(P))^2 \qquad (5)$$

N is the number of data points and P a symbol for the total set of parameters. That set of parameters P_0, which minimizes the function $Chi^2(P)$, is considered as the best possible estimation for the real values.

There are several possibilities to find the minimum of Chi^2. The choice of the optimal search procedure depends on the number of free parameters. Two cases can be distinguished:

Case 1: One-layer walls

Because there are only two parameters (λ and α) to identify, the "exhaustive-search-method" [1] can be used to search the minimum of Chi^2. This method certainly finds the global minimum, since the values of Chi^2 are calculated for all λ-α-pairs with a suitable step wide of λ and α in a specified area.

Case 2: Multi-layer walls

In that case five or more parameters have to be identified. The main problem in finding the minimum of Chi^2 is the great number of local minima in the multidimensional Chi^2-area. The search procedure used has to be robust enough to skip these local minima. For that reason, the "downhill-simplex-method" [1] was chosen and modified for the special application. The modification consists in a three-step-procedure:

Step 1: In the first step the measured data are smoothed before applying the search procedure. This smoothing causes the vanishing of most of the local minima. In that way the procedure reliably finds the global minimum, which can be shifted somewhat in relation to the true one due to the smoothing.

Step 2: The search procedure is applied again using the original data and taking the results from step 1 as initial values.

Step 3: To be sure that the obtained minimum is really the global one as last step a Monte-Carlo-simulation is performed. For that purpose the estimated parameters are varied in a certain range by use of random numbers and the value of Chi^2 is calculated using these parameters. If this is repeated often enough the position of the global minimum can easily be read off from a plot of Chi^2 in dependence on the respective parameters.

The parameters alone do not include information about the quality of the estimation. Therefore it is important to calculate confidence intervals which contain the real values with a statistical certainty of e.g. 95 %. Such a confidence interval is characterized by the fact that the function Chi^2 does not pass over a fixed value Chi^2_{Max} for all values of parameters within the interval. Chi^2_{Max} can be calculated according to [2]

$$Chi^2_{Max} = Chi^2_0 \cdot (1 + F(m,N,s) \cdot m/(N-m)). \qquad (6)$$

In this equation $Chi^2_0 = Chi^2(P_0)$ denotes the minimum of $Chi^2(P)$, m the number of free parameters, s the degree of the statistical certainty and F(m,N,s) the Fischer-distribution, which can be obtained from tables.

From eq. (6) it could be decided that the confidence intervals can be lowered to any degree by an increase of the number of data points, independently whether this increase is realized by an increase in the total duration of the measurement or by a decrease of the measuring interval. However, due to the inertia of every thermal system the additional information get by a repeated decreasing of the measuring interval is limited. In case of too short measuring intervals the data are autocorrelated. This autocorrelation has to be taken into consideration. It can be done by replacing the real number of data points N in eq. (6) by an effective number N_{eff} [3]:

$$N_{eff} = N \cdot (1 - |A|) + \qquad (7)$$

A represents the autocorrelation in the residuals (deviation between calculated and measured heat flow density) over one time step:

$$A = \frac{\sum_{j=1}^{N-1} (\Delta q_j - \overline{\Delta q}) \cdot (\Delta q_{j+1} - \overline{\Delta q})}{\sum_{j=1}^{N} (\Delta q_j - \overline{\Delta q})^2} \qquad (8)$$

$$\Delta q_j = q_{m,j} - q_{c,j} \qquad (9)$$

$$\overline{\Delta q} = \frac{1}{N} \cdot \sum_{j=1}^{N} \Delta q_j \qquad (10)$$

MEASURING EXAMPLES

Case 1: One-layer wall

Fig. 1 shows the results of a measurement at a brick wall (thickness d = 38 cm) and fig. 2 the calculated Chi^2 - area [4]. From its shape the position of the global minimum and hence the values of the parameters can easily be determined.

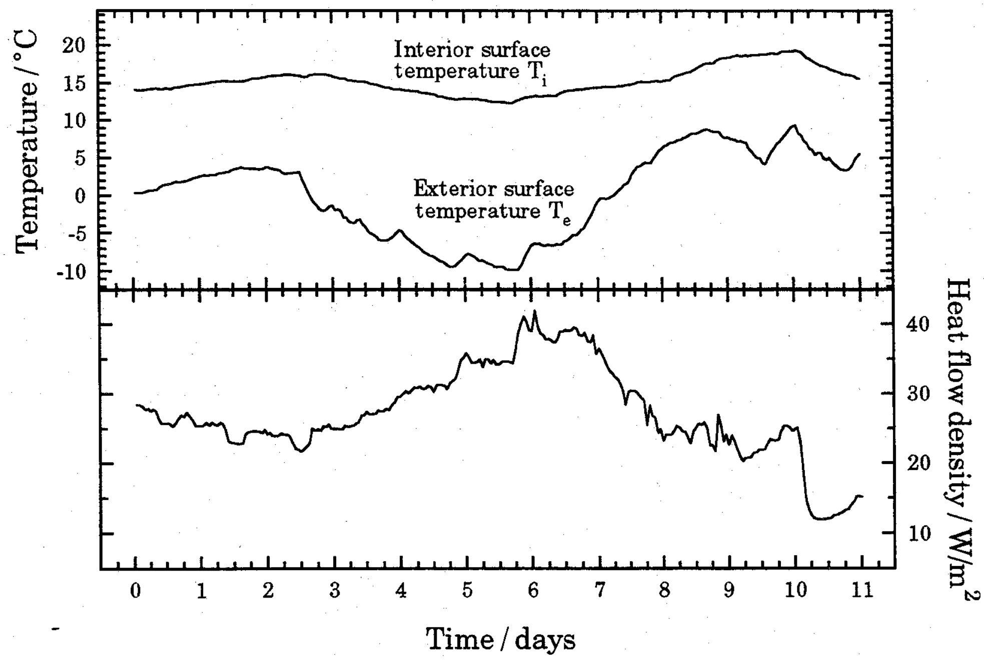

Fig. 1: Data of the one-layer wall

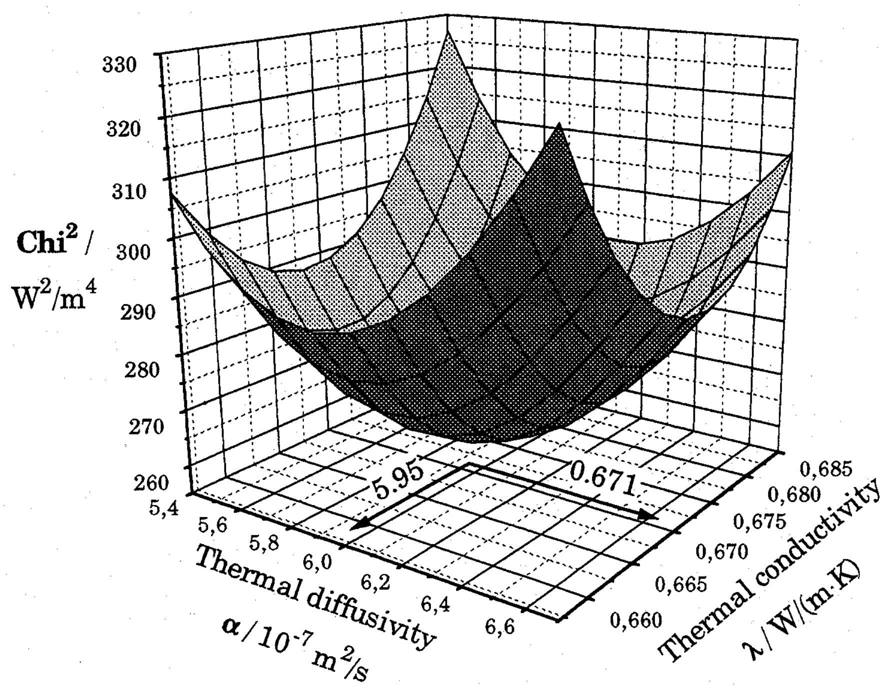

Fig. 2: Chi^2-area of the one-layer wall

The estimation of the confidence intervals is shown in fig. 3. From eq. (6) the value of Chi^2_{Max} for a statistical certainty of 95 % was calculated.

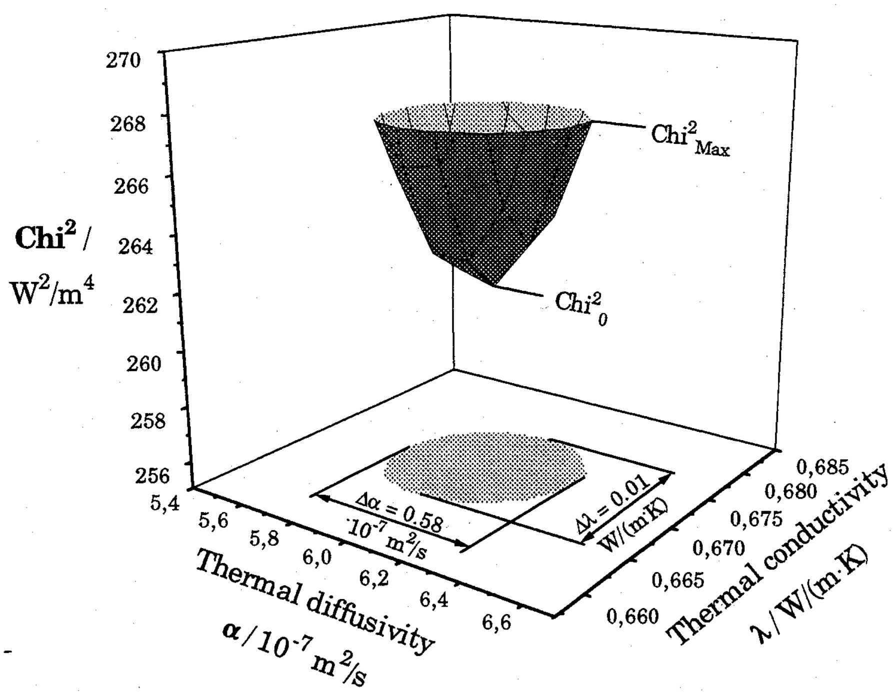

Fig. 3: Determination of the confidence intervals

Another interesting question is the necessary duration of the measurement to obtain a required accuracy of the results. Therefore, the confidence intervals additionally were calculated in dependence on the duration of the measurement. Fig. 4 shows the results in comparison with those obtained by using the well-known averaging method

$$\lambda = d \cdot \frac{\sum_{j=1}^{N} q_j}{\sum_{j=1}^{N} (T_{i,j} - T_{e,j})} \quad (11)$$

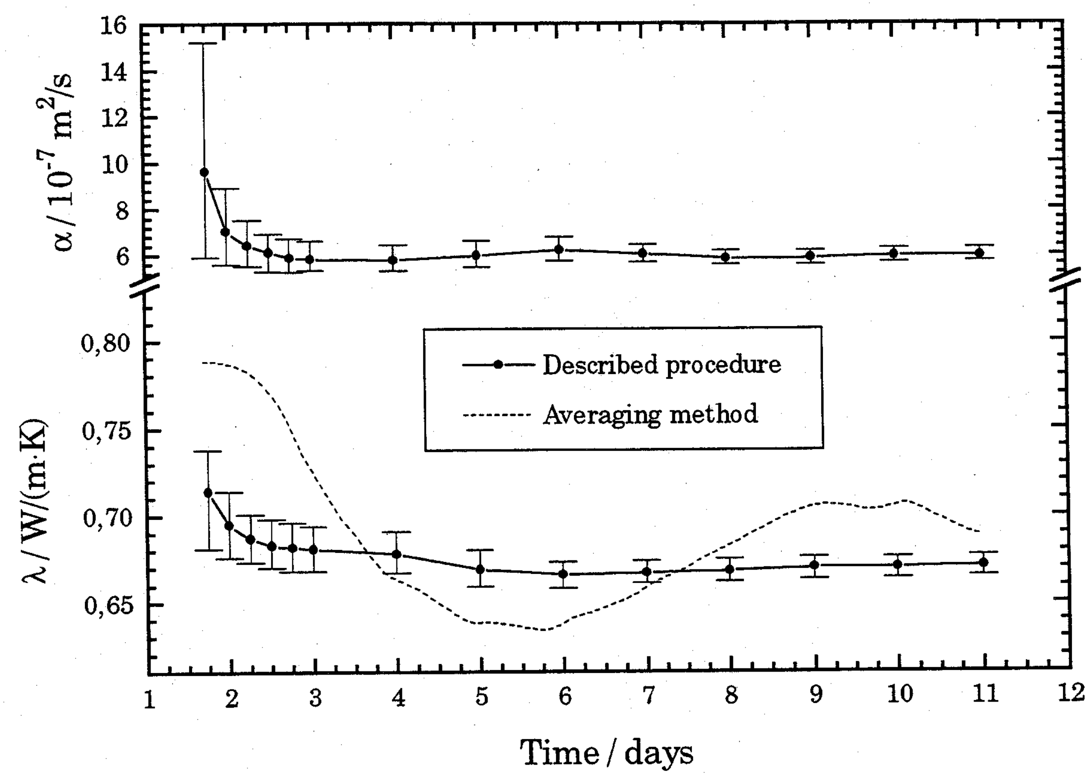

Fig. 4: Results in dependence on the duration of the measurement

It can be seen that the described procedure yields sufficient accurate results already after three days, whereas the analysis by the averaging method does not give a satisfactory result even after 11 days.

Case 2: Two-layer wall

Fig. 5 shows the data from a measurement at a two-layer wall with the following known construction:

- Interior layer: d = 30 cm, $\lambda = 0.800$ W/(m·K), $\alpha = 4.7 \cdot 10^{-7}$ m^2/s
- Exterior layer: d = 10 cm, $\lambda = 0.035$ W/(m·K), $\alpha = 7.2 \cdot 10^{-7}$ m^2/s

(Total thermal resistance: R = 3.232 $m^2 \cdot K/W$)

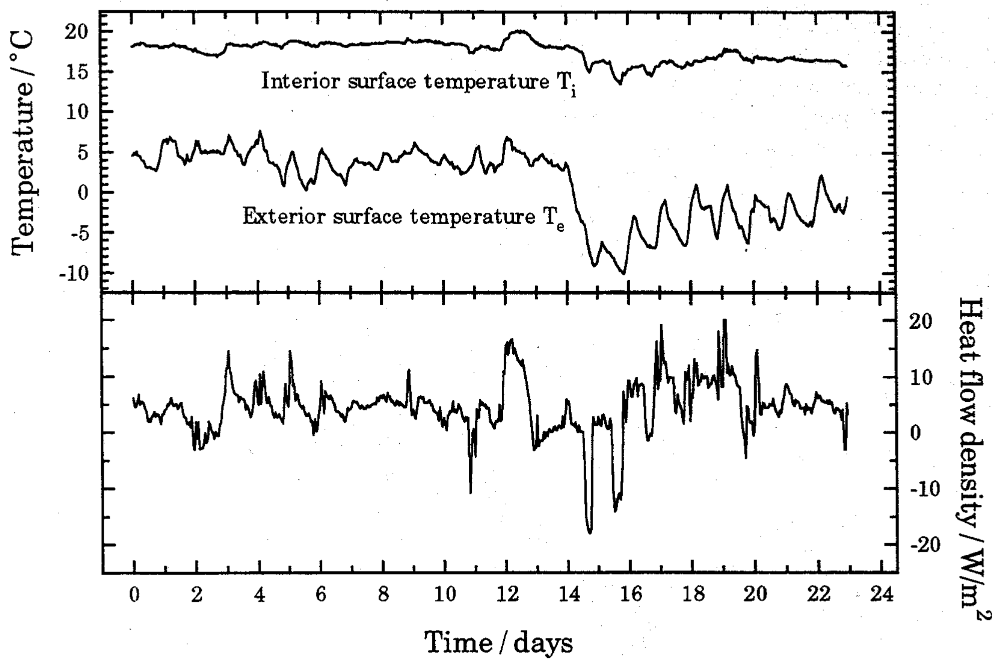

Fig. 5: Data of the two-layer wall

In that case the search for the minimum of Chi^2 was performed by the above-described three-step-procedure. Because in multi-layer walls often the value of the total thermal resistance is of interest, it is useful to plot the calculated Chi^2-values directly against the calculated value of the resistance (fig. 6).

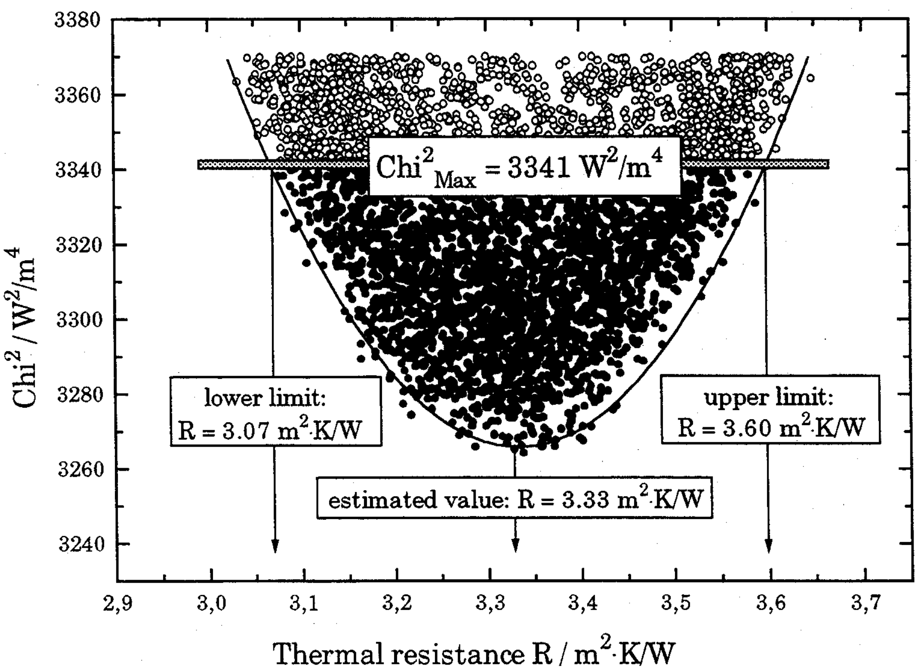

Fig. 6: Chi^2-area calculated by a Monte-Carlo-simulation

Another advantage is that the confidence intervals of the deduced values (R or C) can be taken from this plot, too. In general it is difficult to transform confidence intervals of individual parameters into those of deduced values. The reason is the possible correlation of parameters. It shall be illustrated by an example:
The strong correlation between the thermal conductivity λ_e and the thickness d_e of the exterior layer is shown in fig. 7.

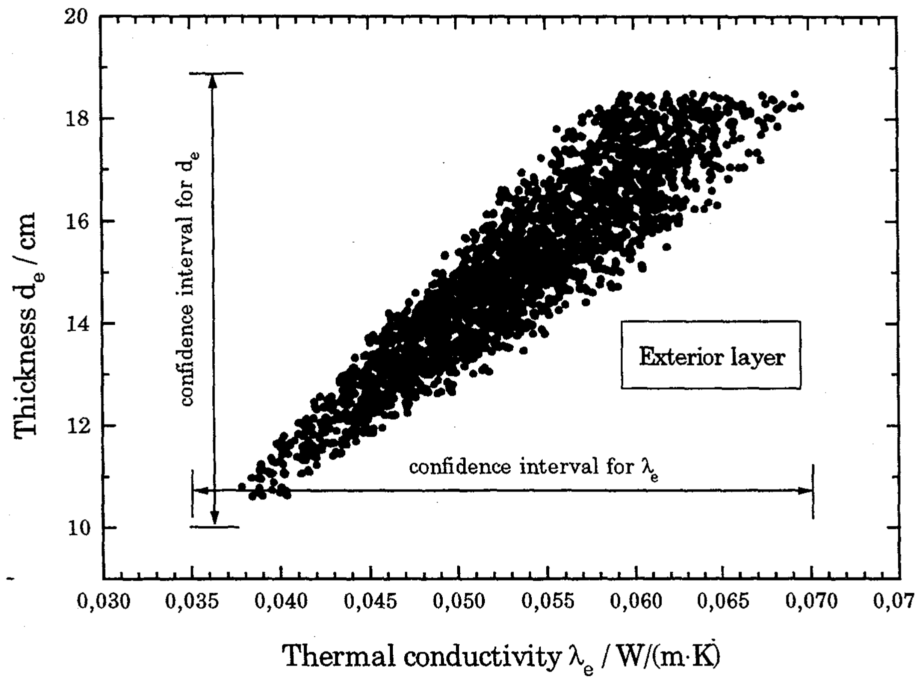

Fig. 7: Correlation between two parameters

It displays the projection of the Chi^2-area within the confidence interval onto the λ_e-d_e-area. These two parameters have relatively large confidence intervals as can be seen in the plot. Therefore, they can be determined only with a large uncertainty. In contrast to this, the thermal resistance of the exterior layer, calculated from these parameters by $R_e = d_e/\lambda_e$, can be determined with a much higher accuracy. The reason is that the points in fig. 7 all lie in a relative narrow band. A high value of λ_e belongs to a high value of d_e and a low value of λ_e belongs to a low value of d_e. Thus the result of the division d_e/λ_e is relative constant.

The following table summarizes the statistical values obtained in the two measuring examples.

	one-layer-wall	two-layer-wall
Number of datapoints N	309	504
Autocorrelation $\|A\|$	0.280	0.059
Effective number of datapoints N_{eff}	223	475
Statistical certainty s	95 %	95 %
Number of parameters m	2	5
Fischer-distribution $F(m,N_{eff},s)$	2.99	2.21
Chi^2_{Max} / Chi^2_0	1.027	1.022

Tab. 1: Statistical values of the two examples

SUMMARY

The described procedure allows to calculate the thermal conductivities, the thermal diffusivities and the thicknesses of individual layers of external walls. The attainable accuracy depends on the quality of the data and on the construction of the wall. Confidence intervals can be calculated for the individual parameters as well as for deduced values. The autocorrelation of data and the correlation between parameters are taken into account.

ACKNOWLEDGEMENTS

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