UNCERTAINTY ANALYSIS IN BUILDING THERMAL MODELLING

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ABSTRACT

This paper introduces a new approach towards the assessment of thermal building performance. As a specific aspect of overall building performance, thermal comfort performance is examined in detail. Two issues have been addressed. Firstly, two comfort performance measures, frequently used in Dutch practice, are analyzed and a directive for the assessment of a better comfort performance measure is discussed. Secondly, a probabilistic simulation experiment has been performed that reveals the adequacy of the present day practice. in which only one year of climate data is used for comfort assessment calculations. It is found that this adequacy depends both on the building under consideration and on the definition of the comfort performance.

INTRODUCTION

In the design stage of a building a designer has to deal with considerable uncertainties. Yet, in thermal performance calculations (of a building under design) these uncertainties are seldom accounted for. A PhD-project has been initiated to determine the main sources of uncertainty, to model these uncertainties and to develop a method to take these uncertainties into account in thermal performance simulations. This paper will report on the first stage. The calculation of thermal comfort for a typical Dutch office building is used as an example to demonstrate how uncertainties can be accounted for in thermal building simulation. The definitions for comfort performance, used by the Dutch Government Building Agency have been analyzed and the role of uncertainties due to the erratic behaviour of outdoor climate on thermal comfort assessment has been investigated.

UNCERTAINTY IN SIMULATION

Building simulations are used to assess the performance of building designs in various stages of the design process. The basis of thermal building simulation consists of solving the temperature field from the heat balance of the building that can approximately be written in the discrete form:

$$\underline{\dot{y}} = \underline{\underline{A}}\,\underline{y} + \underline{\underline{B}}\,\underline{v} + \underline{f} \qquad (1)$$

wherein $\underline{y}$ is the temperature vector, $\underline{v}$ denotes the control vector and $\underline{f}$ is the forcing vector, representing the inputs from outside the system. The matrices $\underline{A}$ and $\underline{B}$ comprise the building model parameters.

Generally, building simulation calculations are carried out deterministically, that is inputs and parameters of the building model are treated as fixed and known values, possibly as a function of time or state. However, the values of these parameters and inputs will often be uncertain. The sources of this uncertainty can be classified in four different categories. These categories will briefly be discussed.

1. Lack of knowledge about the building details in different stages of the design process.
2. Uncertainty due to spread in manufacture and assembly quality of the building components.
3. Unpredictable behaviour of the future users of the building.
4. Lack of knowledge about the underlying physical processes.

This last source of uncertainty affects both the forcing vector and the building parameters. The forcing vector comprises all heat inputs to the system due to heat fluxes from outside the system. These heat fluxes are partly determined by the external climate conditions. It requires no further comment that in this case, due to the complexity of the underlying physical processes, a considerable uncertainty will be introduced. Many building parameters are parametrizations of processes, that are considered to be too complex to model in detail. The simplification in the model structure can introduce an error, the size and sign of which are uncertain.

In present day the designer uses rules of thumb to determine the 'best' values for the model parameters, guided by the brief of the client. As an estimate for the future climate conditions, 'representative' weather data usually for one single year are used. But the designer should be able to calculate the effects of the uncertainties on the results of his performance calculations. This would provide him with a tool to control the reliability of his design.

This paper will deal with thermal building comfort performance as a specific topic of overall thermal building performance.

THERMAL COMFORT

One of the most accepted quantitative models for the assessment of human thermal comfort is the Fanger model [1]. This model is also standardized as a design tool in ISO-7730 [2]. Fanger formulated a quantitative expression for human thermal sensation. This expression is based on a model of the heat exchange between the human body and its environment. According to this expression, the thermal sensation of a subject is determined by subject related parameters as metabolism, clothing level and activity level, and by parameters related to the building(zone) as air temperature, mean radiant temperature, humidity, air velocity. In this paper, the complete set of parameters that determine the thermal sensation will be referred to as the (thermal) state.
Fanger calibrated his expression in several experiments with a vast number of subjects. In these experiments, the subjects were asked to score their sensation of the thermal state on a 7-point scale (vote).

+3	hot
+2	warm
	slightly warm
0	neutral
	slightly cool
-2	cool
-3	cold

Fig. 1. Vote scale, used by Fanger

Fanger based a binary comfort measure on this vote scale. This means that in the Fanger comfort model a subject can experience the thermal state either as satisfying or as dissatisfying (comfortable or not comfortable, 0 or 1). The model is incapable to express the degree of dissatisfaction. Fanger stated that votes with an absolute value less than 2 indicate that the subject is satisfied with the given state, whereas a vote with an absolute value of 2 or more indicates dissatisfaction.

On the basis of his results he derived formula's for two important quantities, the predicted mean vote, PMV and the predicted percentage of dissatisfied people, PPD as a function of the state. The PPD can also be interpreted as an average (dis)comfort sensation (when 0 represents satisfaction and 100% represents dissatisfaction).

COMFORT PERFORMANCE

The Fanger model provides for a tool to calculate the average comfort sensation for a given state. The issue, however, is to assess the comfort performance of a building. The building comfort performance is an index that indicates how good or how bad a building is with respect to the (thermal) comfort. The avarage comfort sensation as obtained by the Fanger model is inadequate as a building performance index for two reasons.
First, the average comfort sensation reflects the state of mind of (future) users, whereas the comfort performance should express the state of mind of the decisionmaker(s) in the design process. Therefore, this decisionmaker should assign quantitative preferences to all possible values of the (average) comfort sensation. These preferences can be expressed in a comfort performance function, that assigns a comfort performance value (preference) to every possible comfort sensation, that is to every possible state. We refer to this comfort performance value as a *state* comfort performance.
This state comfort performance is still not a suitable measure for the comfort performance of the building, as it is time dependent. A decision must be made to determine how the building comfort performance should be defined in terms of the state comfort performance to reach a time independent index.

Two definitions for building comfort performance, used in The Netherlands are discussed here.

In The Netherlands, there are no directives for building comfort performance imposed by law, but the client's brief will usually require strict limits on building comfort performance. For example, buildings that are designed in commission of the Government Building Agency (GBA) have to comply with the internal GBA-standards [3]. These

GBA standards are based on the recommendation in annex D of the ISO-7730 standard:

In this annex, thermal comfort requirements are recommended for spaces for human occupancy. It is recommended as acceptable that the PPD be lower than 10%.

This recommendation can be interpreted as a preliminary *state* comfort performance function. But the formulation is vague, qualitative and does not give a clue about situations that the PPD exceeds 10%.

In the first GBA-standard, this qualitative recommendation was interpreted as a binary state comfort performance function: in situations, that the PPD is less than 10%, the state comfort performance is O.K., and in situations that the PPD exceeds 10%, the state comfort performance is not O.K. The GBA decided to define the building comfort performance as the state comfort performance, integrated over one year. The GBA imposes a lower limit on this building comfort performance.

In the second GBA-standard the notion was incorporated, that it appears logical to assume that the state comfort performance not (only) depends on whether or not the PPD exceeds 10%, but (also) on the degree of discomfort (the PPD). This GBA standard uses the following state comfort performance function:

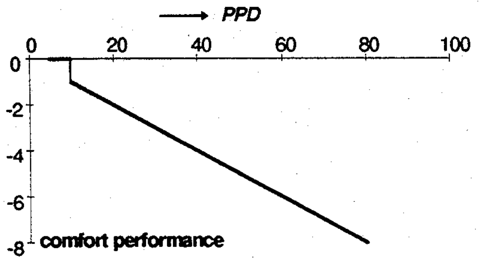

Fig. 2. State comfort performance function, used in the most recent internal GBA-standard. The negative sign indicates that the comfort performance decreases as the PPD increases.

Again, in the GBA-regulations, the building comfort performance is the state comfort performance, integrated over 1 year. As in the first standard, the GBA imposes a lower limit to this building comfort performance.

Both GBA-definitions of the state comfort performance function for PPD-values over 10% do not go beyond the level of common-sense reasoning. However, common-sense reasoning will not lead us much further than the conclusion that (probably) state comfort performance will decrease as the discomfort level increases. Other features, for example the most recent state comfort performance function used by the GBA, cannot be deduced by common sense. For example, the height of the step at PPD=10% is chosen arbitrarily as is the linear relationship between the state comfort performance and the PPD.

The process that leads to the assessment of a proper state comfort performance definition should contain the following stages:

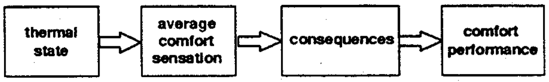

Fig. 3. Process that leads to a proper assessment of a comfort performance function.

The state comfort performance should indicate how good or how bad a given comfort sensation is. Therefore an assessment of the consequences of that comfort sensation is required (e.g. loss of productivity, ailment, dissatisfaction, etc.). These consequences will generally be of dissimilar nature and partly imponderable. The issue to come to the assignment of numerical values to the consequences, that is the definite assessment of the state comfort performance function, will generally be a matter of policy makers and regulatory bodies.

PROBABILISTIC APPROACH

The uncertainty in the data and the model structure to be used by the designer, as mentioned earlier in this paper, will lead to uncertainty in the comfort performance of the future building. How should a designer make decisions on the basis of this uncertain performance?

If the building comfort performance function properly expresses the preferences or values of the consequences as a function of the comfort level, it can be demonstrated [4] that the expected or average building comfort performance is a logical basis for decision. By modelling the uncertainties as stochastic properties, the expected building comfort performance can be calculated with the tools of stochastic and statistic theory.

To gain insight in the behaviour of the expected comfort performance, it is useful to introduce some mathematics.

In the GBA regulations, the building comfort performance is defined as the state comfort performance, integrated over 1 year:

$$U_B = \int_T U_S \, dt \tag{2}$$

where:
U_B : building comfort performance
U_S : state comfort performance
T : time period with the length of 1 year

As the state performance is a random variable, expressing the uncertainty in its value, the building comfort performance is also a random variable. The expected building comfort performance, that is required as a basis for decision, can be written as:

$$E\{U_B\} = \int_T E\{U_S\} \, dt \tag{3}$$

By definition:

$$E\{U_S\} = \int_{-\infty}^{0} \xi \, f_{U_S}(\xi) \, d\xi \tag{4}$$

with:
$E\{.\}$: the expected value
$f_{U_S}(\xi)$: the probability density for U_S.

If the state comfort performance is a function, defined on the PPD from the Fanger model, (4) can be written as:

$$E\{U_S\} = \int_{5\%}^{100\%} u_S(\varphi) \, f_{PPD}(\varphi; t) \, d\varphi \tag{5}$$

with $f_{PPD}(\varphi;t)$ the probability density for the PPD.

Due to the yearly cycle in the outdoor climate, the probability density for the PPD is time dependent with also a period of 1 year.

By substituting (5) into (3) we obtain:

$$E\{U_B\} = T \int_{5\%}^{100\%} u_S(\varphi) \left\{ \frac{1}{T} \int_T f_{PPD}(\varphi; t) \, dt \right\} d\varphi$$

$$= T \int_{5\%}^{100\%} u_S(\varphi) \, \overline{f_{PPD}}(\varphi) \, d\varphi \tag{6}$$

where:

$\overline{f_{PPD}}(\varphi)$ the time averaged probability density for the PPD over 1 year.

Unless stated otherwise, this time averaged probability density for the PPD will be referred to simply as the probability density. Equation (6) points out, that a proper estimate for the PPD probability density is the basis for a probabilistic assessment of the expected building comfort performance.

The main issue in the ongoing PhD-project will be the question: how closely can the expected comfort performance of a building be approximated by using rules of thumb and 'best' values in stead of a probabilistic approach? In which cases this will not be sufficient? In this paper this issue is addressed with respect to the uncertainties arising from the erratic behaviour of the climate.

SIMULATION EXPERIMENT

A substantial contribution to the uncertainty in the future building performance arises from the erratic behaviour of the external climate. In present day in The Netherlands, a designer generally uses the climate data of 1964 or the 'artificial' Dutch Test Reference Year to calculate the future comfort performance of a building under design. The question is how adequate this comfort performance is as an estimate for the expected comfort performance.

From (6) follows, that in general, a proper estimate for the expected comfort performance can be assessed only if the probability density for the PPD can be estimated properly.

To determine if a proper estimate for the probability density for the PPD can be assessed on the basis of the climate data of one year only (1964 or the Test Reference Year), a simulation experiment was performed.

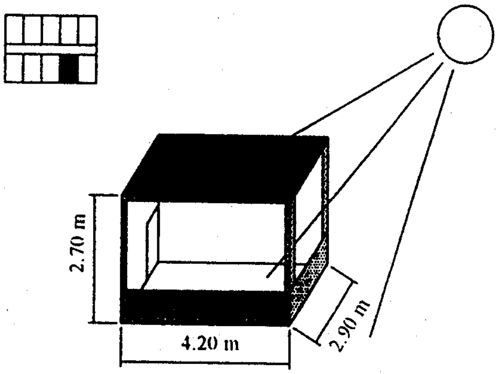

Fig. 4 Basic office room variant with default parameters used in the simulation experiment

This simulation experiment focussed on the loss of comfort performance due to overheating problems only. Thermal simulations were carried out for 27 variants of an office room in a standard office building layout. The basic variant with default parameter values is sketched in fig. 4.
The 27 variants were formed by making all possible combinations of three different values for three characteristics:

Table 1. Characteristics of the office room that were varied in the simulation study, and their values.

characteristic	values
orientation	west, south, east
building mass (depth of the room)	0.67, 1.0, 1.5 times 4.20 m
glass percentage of facade	20%, 40%, 60%

Consistent with the latest GBA-regulations, no cooling equipment was modeled in the building, necessitating an assessment of summer overheating risk. A shading device was assumed, triggered by the solar irradiance level. In each variant the internal heat load was adapted, so that the comfort performances, calculated with the climate data of 1964 and the most recent GBA-comfort performance function (fig. 2) came close to the allowed minimum value (-150 hours) The simulation was done with a dedicated configuration of the finite element toolbox BFEP [5].

The simulations were carried out with 10 years of real climate data and the Dutch Test Reference Year. For all years an estimate for the probability density for the PPD as defined in eqation (6), was calculated. The best estimate for the probability density was obtained by averaging the probability densities over the years 1961-1970. The probability that the PPD is in a certain interval was estimated by the percentage of the total time that the simulated PPD was in that interval.

RESULTS

Three characteristic simulation results are shown in fig. 5, 6 and 7. In stead of the probability densities, the complementary cumulative probability distributions are plotted. This distribution gives for every value φ in the PPD domain the probability that the PPD will exceed this value. The building variants shown in fig. 5, 6 and 7 all have a west orientation, but the results for other orientations were similar.

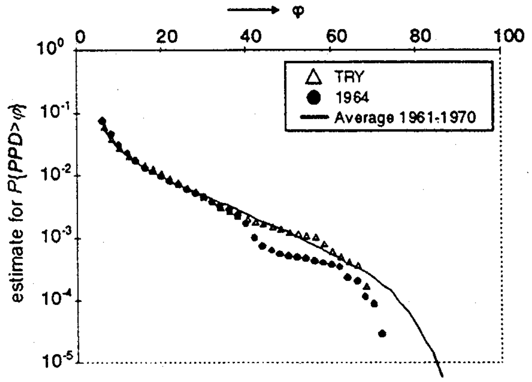

Fig. 5. Estimates for the complementary cumulative PPD-distribution for office room variant: orientation west, depth 2.80 m, glass percentage of facade 60%.

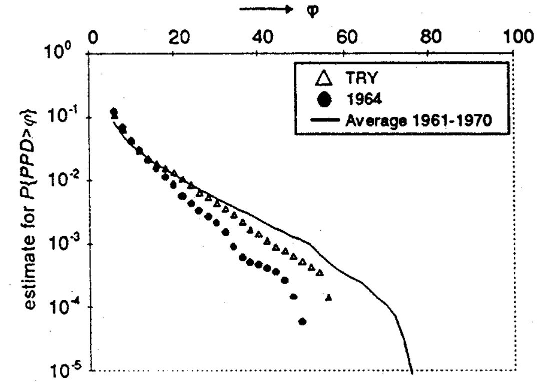

Fig. 6. Estimates for the complementary cumulative PPD-distribution for office room variant: orientation west, depth 6.30 m, glass percentage of facade 60%.

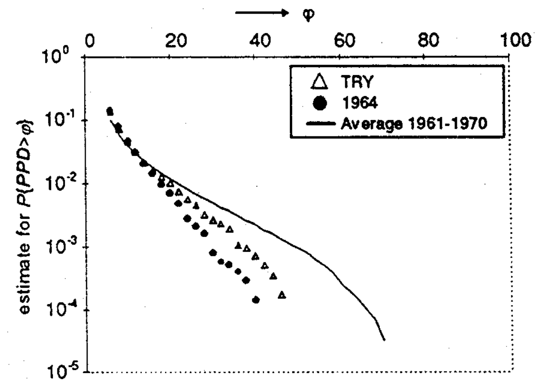

Fig. 7. Estimates for the complementary cumulative PPD-distribution for office room variant: orientation west, depth 6.30 m, glass percentage of facade 20%.

In table 2 the expected building comfort performance is listed for these 3 variants and for two comfort performance functions. Comfort performance function number 1 and number 2 are respectively the first GBA comfort performance and the latest GBA comfort performance, both mentioned before.

Table 2. Expected comfort performances for three variants of the office room (in hours). For each variant, the expected building comfort performance is listed for two different comfort performance functions.

	office room variant:			
	orientation:	west	west	west
	depth:	2.80 m	6.30 m	6.30 m
	glazing percentage:	60%	60%	20%
comfort performance function	*simulation period*			
1	1964	-76	-100	-102
1	TRY	-70	-102	-106
1	average 1961-1970	-68	-100	-102
2	1964	-154	-165	-158
2	TRY	-158	-198	-181
2	average 1961-1970	-154	-209	-203

DISCUSSION

From fig. 5 it is obvious, that for the building variant with a large glass fraction of the facade and small mass, the PPD probability density estimate for 1964 and the Test Reference Year agrees with the average probability density rather well. As the mass increases and/or the glass percentage decreases, this agreement gets worse. Fig. 6 and fig. 7. illustrate this effect.

This is not surprising, since both decreasing the glass area and increasing the building mass make the PPD-fluctuations slower and more auto-correlated in time. This means that the simulation results over 1 year contain less statistically independent information, which leads to an increase in the variance of the simulation results. As a consequence, the probability that the results from a simulation over a period of 1 year are close to the average result is smaller (law of large numbers).

But as is illustrated by equation (6), it is not only the probability distribution for the PPD that determines the expected comfort performance, also the choice of the state comfort performance function is important. Table 2 shows, that for state comfort performance function 1 that does not depend on the exact course of the PPD distribution, one can rely on 1964 or Test Reference Year to give results that are close to the long term estimate. But if the dependency on the PPD distribution becomes a factor (state comfort performance function 2), the simulation results for one single year do no longer a priori give a valid estimate for the expected building comfort performance. The validity then depends on the building under consideration.

CONCLUSIONS

A probabilistic approach towards the assessment of building thermal performance has been discussed. This approach accounts for the uncertainties in model parameters and model inputs in thermal building simulation. The sources of uncertainty have been discussed. Thermal building comfort performance has been examined in detail as a specific topic of thermal building performance. Two issues were addressed.

Firstly, two definitions of building comfort performance, frequently used in Dutch practice, were analyzed. It appears that these definitions are based on common-sense reasoning rather than on proper assessment of the consequences occurring from thermal (dis)comfort.

Secondly, the present day practice of building comfort performance assessment with only the climate data of 1964 has been evaluated probabilistically. A simulation experiment was performed, in which the comfort performance of 27 building variants was calculated, both for 1964 and for a period of 10 years. It was found that for the building variants with a relatively large glazing percentage of the facade and low mass, the 1964 estimates are close to the 10 year averages. For variants with larger building mass and/or a smaller glazing percentage, this agreement deteriorates. The degree of deterioration depends on the definition of the comfort performance.

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NOMENCLATURE

$\underline{x}$	vector
$\underline{X}$	matrix
$\boldsymbol{X}$	random variable
$E\{\boldsymbol{X}\}$	expected value of $\boldsymbol{X}$
f_X	probability density function of $\boldsymbol{X}$
$P\{\boldsymbol{X}>\xi\}$	probablity that $\boldsymbol{X} > \xi$
$\underline{v}$	temperature vector: discrete spatial distribution of the temperature in the simulated system (building)
	control vector in the simulated system
	forcing vector on the simulated system
U_B	building comfort performance
U_S	state comfort performance
t	time
T	time period with length of 1 year.
PPD	predicted percentage of dissatisfied people according to Fanger