BUILDING SIMULATION IN A MATHEMATICAL PROGRAMMING ENVIRONMENT

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ABSTRACT

A new approach for computer-aided thermal analysis of buildings is presented. It is based on an electronic book "Building Thermal Analysis" which operates in a mathematical programming environment (Mathcad). It consists of a set of independent files covering various topics of building thermal analysis linked together with a hypertext system. Each file integrates "live" equations (the "program "), explanatory text and figures, as we11 as graphs linked dynamically to equations and data. Both equations and data can be readily changed and the results are instantly updated within the same document, thus providing unique educational and rescarch capabilities. The main advantage over the use of "black box" building the~ analysis software is that the user consolidates in a single document the input data, the output data, and the model, thus having the capability to easily understand and change the model as we11 as the data.

INTRODUCTION

In the last decade mathematical programming environments such as Mathcad and Mathematica have become available on microcomputers, and have found widespread use for teaching, research, and design. This paper describes an innovative approach for building thermal analysis using a new electronic book [1] within a high level programming mathematical environment, namely Mathcad [2].

Mathcad is a high level programming environment which operates within Microsoft Windows, Unix, and other operating systems. It can integrate "live" equations (the "program"), explanatory text and figures, as well as text and graphs linked dynamically to equations and data which are in the same file.Both equations and data can be readily changed and the results are instantly updated within the same document, thus providing unique educational and research capabilities. Mathematical options available include solutions of non-linear equations used in psychrometric property calculations, matrix algebra which is used for example in finite differece solution of a thermal bridge model, and complex variables used in wall thermal admittance calculations. Figure 1 lists the contents of "Building Thermal Analysis", each topic represented by files which are linked together with a hypertext system, and a hypertext index.

The electronic book has been used for short courses in heating and cooling analysis and design, addressed to heating and airconditioning engineers. The advantage over the use of building energy analysis programs is that the user has in one document the input data, the output data, and the model, thus having the capability to understand and change the model.

TECHNIQUES OF BUILDING THERMAL ANALYSIS

Building thermal analysis is generally associated with heating and cooling system design, building envelope heat transfer calculations, thermal comfort calculations and other related topics. Some of the most important objectives of building thermal analysis are building envelope thermal optimization and sizing of heating and cooling equipment and terminal heat distribution devices to maintain thermal comfort conditions in each zone of a building. Building thermal analysis is generally taught as part of a heating-ventilation-air-conditioning course in the final year of Mechanical Engineering undergraduate programs (e.g. University of Alberta in Canada [3]) or sometimes as a specialized course in Building Engineering academic programs (e.g. Concordia University in Canada [4]). Also, a major part of building thermal analysis is taught in renewable energy courses at the graduate level or senior undergraduate level in Engineering and Applied Physics programs (e.g. Applied Physics, University of Athens, Greece). Mechanical Engineering programs which graduate most HVAC engineers tend to focus on the equipment side of building thermal design at the expense of thermal design of the building envelope. At the same time, architects typically make the critical decisions for the building envelope such as percent window area and type, and wall construction at the early design stage. The majority of architects usually lack any indepth background in building thermal analyis and thus typically make critical decisions about the building envelope based on non-thermal criteria - usually easthetic, appearance etc. Consequently, the building envelope is rarely thermally optimized.

Numerous computer simulation programs have been developed for building energy analysis such as DOE [5] and TRNSYS [6]. Such programs have, to a large extent, replaced handbooks as tools for thermal design. Also, many simplified programs are simply computer implementations of handbook methods such as the CLTD method [7]. Building energy analysis software have contributed tremendously to advancing the precision, quality and productivity of building and HVAC system thermal design. However, with the exception of a few open architecture programs such as TRNSYS [6] they are essentially black boxes providing the user with the capability to specify inputs and sometimes outputs based on a set of predefined mathematical models of building and equipment components; usually, these models cannot be changed, thereby reducing the flexibility of design. Thus, a major disadvantage of such software is that they are often used for conditions for which they are not valid, or their results are misinterpreted due to poor understanding of the mathematical models on which they are based.

Similar problems are encountered in other areas of engineering design when using simulation software generally developed with procedural high level computer languages such as FORTRAN. In an attempt to address this problem, higher level flexible mathematical programming environments such as Mathcad and Mathematica have become available on small computers, and are increasingly being used for teaching, research, and design. "Building thermal analysis" is an electronic book developed within Mathcad to provide a flexible tool for teaching, research and design in this area. One of the main advantages of this approach is that the user can see, and modify, the input data, the model and explanatory text, plus graphs linked to equations, all in one document (file). Figure 1 lists the contents of "Building Thermal Analysis", each topic represented by files which are linked together with a hypertext system, and a hypertext index.

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BUILDING SIMULATION IN A MATHEMATICAL PROGRAMMING ENVIRONMENT

"Building thermal analysis" [1] covers heat transfer in buildings, with emphasis on building envelope thermal calculations. This is extremely important and necessary when designing a passive solar building or a house which will be cooled with natural ventilation [6]. The electronic book [1] starts with steady-state situations of mixed mode heat transfer such as thermal resistance calculation for walls and pipes, followed by simple analysis of transient heat conduction based on semi-infinite solid models and the thermal network method (see Figure 1).

The finite difference method is then introduced in

BUILDING THERMAL ANALYSIS

1. **Steady-state heat conduction in multi-layered walls and pipes**
    - 1.1 Thermal resistance calculation methods for walls and pipes
    - 1.2 Walls with internal heat generation (e.g. radiant panels)
    - 1.3 Conduction shape factors

2. **Transient heat conduction in buildings**
    - 2.1 Lumped parameter systems and the thermal network method
    - 2.2 Semi-infinite solids - e.g. pipe buried in soil
    - 2.3 Simple model for a wall

3. **Analysis of heat conduction in walls with the finite difference method**
    - 3.1 Steady-state two-dimensional example; thermal bridges
    - 3.2 Heat flow in basements
    - 3.3 Transient one-dimensional finite difference analysis of wall heat flow

4. **Periodic heat flow in multi-layered walls**
    - 4.1 Thermal admittance and impedance of walls
    - 4.2 Steady-periodic heat transfer in multi-layered walls

5. **Convection and infiltration in rooms and cavities**
    - 5.1 Natural convection in cavities (windows etc.)
    - 5.2 Forced convection heat transfer coefficients
    - 5.3 Infiltration

6. **Radiation heat transfer in buildings**
    - 6.1 Calculation of view factors in a rectangular room with one window
    - 6.2 Emittance and absorbtance calculations
    - 6.3 Combined radiation and convection

7. **Solar radiation**
    - 7.1 Solar radiation incident on inclined surfaces (walls, roofs, windows)
    - 7.2 Solar properties of windows
    - 7.3 Solar radiation transmitted through windows
    - 7.4 Solar shading calculations

8. **Psychrometrics and thermal comfort**
    - 8.1 Thermodynamic properties of moist air
    - 8.2 Thermal comfort calculation

9. **Heating and cooling load calculations with transient models for a zone**
    - 9.1 First-order room transient model
    - 9.2 High-order steady-periodic model for a zone
    - 9.2 Sensible heating/cooling load and latent cooling load

10. **Building thermal control**
    - 10.1 Laplace transfer functions
    - 10.2 Z-transfer functions
    - 10.3 PID control of a heating system

Figure 1 Contents of Building Thermal Analysis chapter 3 and applied to two-dimensional steady state heat conduction problems, such as thermal bridge heat loss calculation for a balcony and a basement. A transient finite difference model is then presented for a wall with transparent insulation. The thermal admittance method using complex variables and Fourier series is used for steady-periodic heat flow calculations in chapter 4, followed by calculation of convective heat transfer coefficients in chapter 5. Chapter 6 determines radiation view factors for a room and combined radiation-convection heat transfer coefficients. Chapter 7 calculates instantaneous solar radiation incident on inclined surfaces, and transmitted through single-glazed and double-glazed windows; a typical example determines optimum tilt angle for a solar collector. Chapter 8 determines psychrometric properties of moist air and thermal comfort. Chapter 9 determines heating and cooling loads for a zone using Fourier series for weather inputs and outputs such as instantaneous auxiliary heating load; the wall thermal admittance transfer functions described in chapter 4 are used to determine zone transfer functions. Finally, chapter 10 introduces thermal control analysis of feedback loops in HVAC systems using Laplace transfer functions and numerical inversion of the Laplace domain response to the time domain, followed by a z-transform model of a digital PID (proportional-integral-derivative) loop.

Figures 2a-2b illustrate thermal analysis of a double-glazed window with and without low-emissivity coating. Note that equations and text appear in different font (and different colour on a monitor). The electronic book has been used for short courses on heating and cooling analysis and design, addressed to heating and air-conditioning engineers. Feedback from the participants is that they generally found it an effective interactive computerized learning tool. On the other hand, if the user is not disciplined, and if the model in a file is changed without careful analysis of the effects of the changes, erroneous results may be obtained. Also, on computers with microprocessors slower than an INTEL 80486, large documents will run slowly.

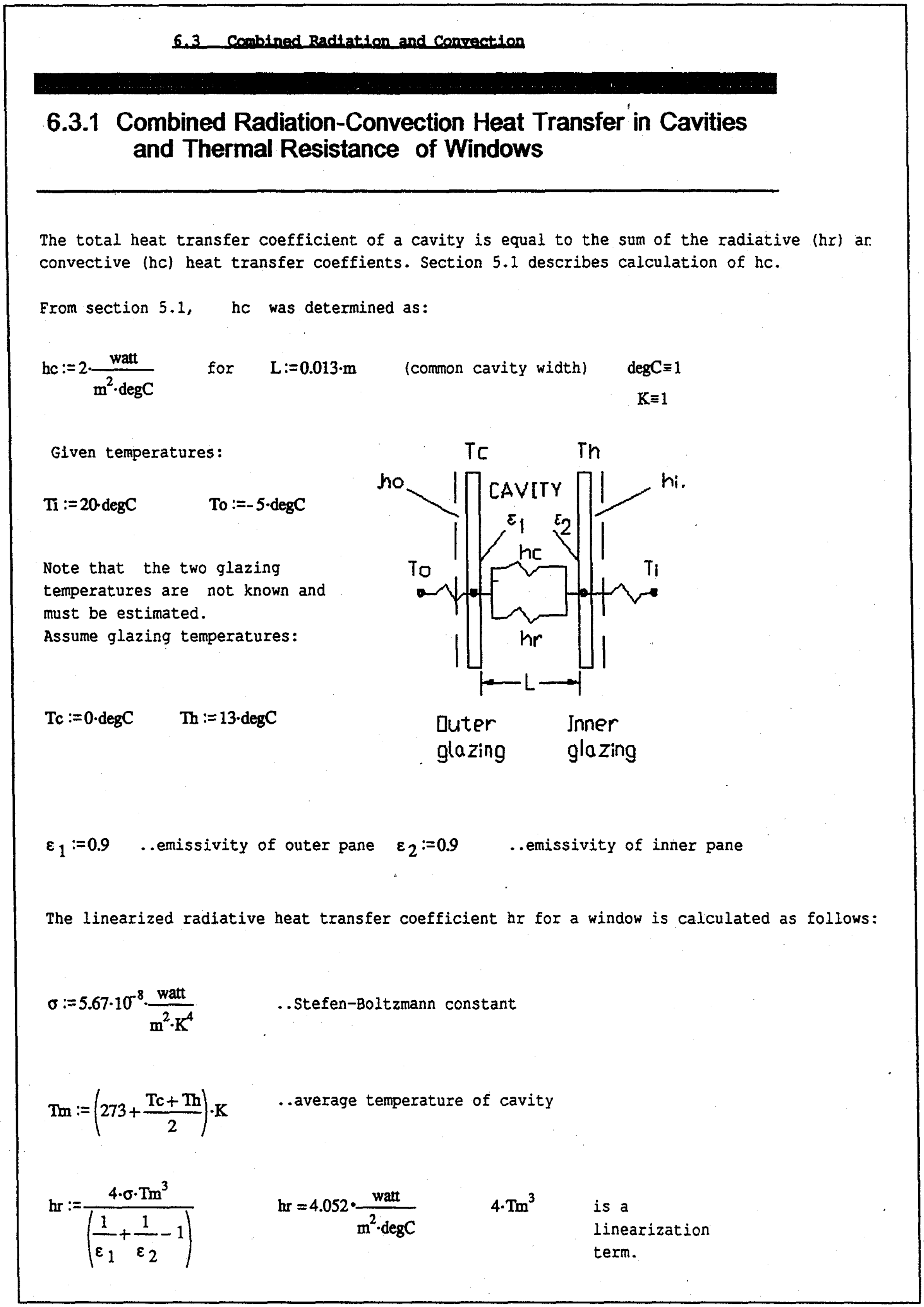
CONCLUSION

A new approach for building thermal analysis using an electronic book within a high level programming environment was presented. It integrates "live" equations (the "program"), explanatory text and figures, as well as text and graphs linked dynamically to equations and data which are in the same file. Both equations and data can be readily changed and the results are instantly updated within the same document, thus providing unique educational and research capabilities.

The electronic book has been used for short courses in heating and cooling analysis and design, addressed to heating and air-conditioning engineers. The main advantage over the use of building energy analysis programs is that the user consolidates in a single document the input data, the output data, and the model, thus having the capability to easily understand and change the model as well as the data.

REFERENCES

1. A.K. Athienitis, 1993, "Building Thermal Analysis", Mathcad electronic book, Pub. MathSoft Inc., Boston Mass., USA.
2. Mathcad 3.1, 1993, MathSoft Inc., Boston Mass., USA.
3. The University of Alberta, 1993, Undergraduate Calendar, Edmonton, Alberta, Canada.
4. Concordia University, 1993, Undergraduate Calendar, Montreal, Quebec, Canada.
5. DOE-2 , Reference Manual 2.1A, 1981, Los Alamos Scientific Laboratory.
6. S. Klein et al., 1988, "TRNSYS - A Transient System Simulation Program", University of Wisconsin-Madison, Engineering Experiment Station Report 38-12.
7. ASHRAE, 1989, ASHRAE Handbook-Fundamentals, Atlanta, Georgia, USA.
8. M. Santamouris and D.N. Asimakopoulos (editors), 1994, "Passive Cooling of Buildings", European Comm. Directorate General for Energy, C.I.E.N.E., Athens, Greece.



6.3 Combined Radiation and Convection

6.3.1 Combined Radiation-Convection Heat Transfer in Cavities and Thermal Resistance of Windows

The total heat transfer coefficient of a cavity is equal to the sum of the radiative (hr) ar convective (hc) heat transfer coeffients. Section 5.1 describes calculation of hc.

From section 5.1, hc was determined as:

$hc := 2 \cdot \frac{watt}{m^2 \cdot degC}$ for $L := 0.013 \cdot m$ (common cavity width) $degC \equiv 1$ $K \equiv 1$

Given temperatures:

$Ti := 20 \cdot degC$ $To := -5 \cdot degC$

Note that the two glazing temperatures are not known and must be estimated.
Assume glazing temperatures:

$Tc := 0 \cdot degC$ $Th := 13 \cdot degC$

$\varepsilon_1 := 0.9$ ..emissivity of outer pane $\varepsilon_2 := 0.9$ ..emissivity of inner pane

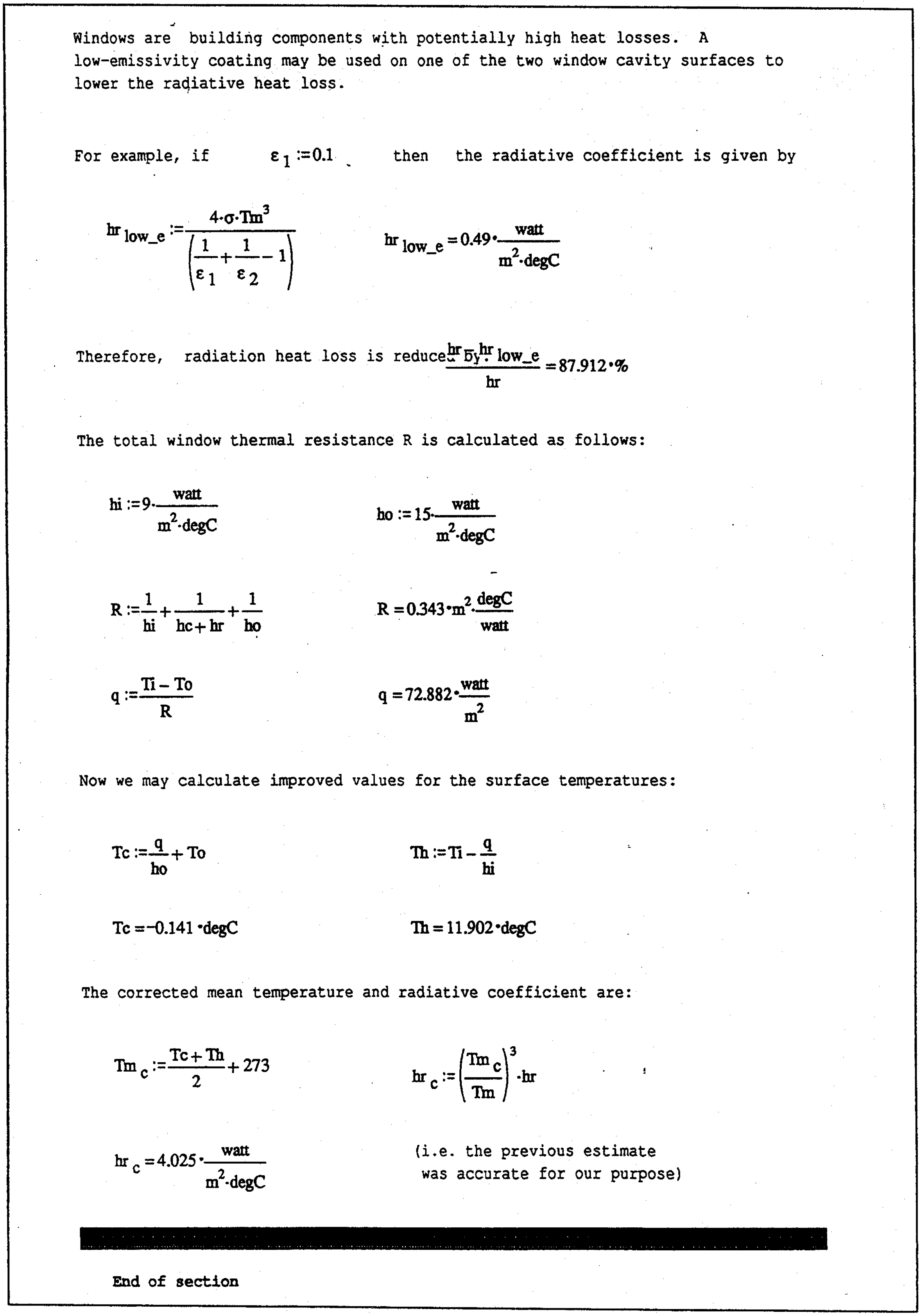
The linearized radiative heat transfer coefficient hr for a window is calculated as follows:

$\sigma := 5.67 \cdot 10^{-8} \cdot \frac{watt}{m^2 \cdot K^4}$ ..Stefen-Boltzmann constant

$Tm := \left(273 + \frac{Tc + Th}{2}\right) \cdot K$ ..average temperature of cavity

$hr := \frac{4 \cdot \sigma \cdot Tm^3}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$ $hr = 4.052 \cdot \frac{watt}{m^2 \cdot degC}$ $4 \cdot Tm^3$ is a linearization term.

Figure 2a. Example document from "Building Thermal Analysis"



Windows are building components with potentially high heat losses. A low-emissivity coating may be used on one of the two window cavity surfaces to lower the radiative heat loss.

For example, if $\varepsilon_1 := 0.1$ then the radiative coefficient is given by

$$hr_{low\_e} := \frac{4 \cdot \sigma \cdot Tm^3}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} \qquad hr_{low\_e} = 0.49 \cdot \frac{watt}{m^2 \cdot degC}$$

Therefore, radiation heat loss is reduced by $\frac{hr - hr_{low\_e}}{hr} = 87.912 \cdot \%$

The total window thermal resistance R is calculated as follows:

$$hi := 9 \cdot \frac{watt}{m^2 \cdot degC} \qquad ho := 15 \cdot \frac{watt}{m^2 \cdot degC}$$

$$R := \frac{1}{hi} + \frac{1}{hc + hr} + \frac{1}{ho} \qquad R = 0.343 \cdot m^2 \cdot \frac{degC}{watt}$$

$$q := \frac{Ti - To}{R} \qquad q = 72.882 \cdot \frac{watt}{m^2}$$

Now we may calculate improved values for the surface temperatures:

$$Tc := \frac{q}{ho} + To \qquad Th := Ti - \frac{q}{hi}$$

$$Tc = -0.141 \cdot degC \qquad Th = 11.902 \cdot degC$$

The corrected mean temperature and radiative coefficient are:

$$Tm_c := \frac{Tc + Th}{2} + 273 \qquad hr_c := \left(\frac{Tm_c}{Tm}\right)^3 \cdot hr$$

$$hr_c = 4.025 \cdot \frac{watt}{m^2 \cdot degC}$$

(i.e. the previous estimate was accurate for our purpose)

End of section

Figure 2b. Example document from "Building Thermal Analysis" (continued from 2a)