THE THEORY OF PLUMES ADAPTED TO MODEL AIR MOVEMENT IN NATURALLY VENTILATED BUILDINGS

Paul Cooper

Department of Mechanical Engineering
University of Wollongong,
Wollongong 2522, Australia

ABSTRACT

This paper describes ongoing research toward the development of simplified techniques for the prediction of air movement in large, naturally ventilated spaces containing hot and/or cold surfaces. The situation where two distinct sources of heat are present on the floor of a naturally ventilated room is discussed Thermal stratification develops such that two layers of warm air form above a lower layer at ambient temperature. The heights and temperatures of these layers are predicted using forced plume theory, Le. where a plume is initiated from a source of buoyancy. with finite mass and momentum flux). Interface heights are found to depend on room height, ventilation opening size and on the ratio of the heat fluxes from the two sources. The intensities of the heat sources have no influence on interface heights. A theoretical and experimental investigation of a naturally ventilated enclosure containing both positive and negative sources of buoyancy is also presented Large glazed spaces such as atria exhibit significant thermal stratification due to radiative heating of internal surfaces. The possibility of extending the plume analysis for isolated sources of buoyancy in a space to that where the sources of buoyancy are distributed is discussed

INTRODUCTION

Natural ventilation of spaces within buildings has always formed an important part of architectural design and is becoming ever more important as it is seen as both a means of saving energy and a way of improving occupant perceptions of the building (as compared to mechanically ventilated spaces). Knowledge of the thermal characteristics of naturally ventilated spaces and associated air movement is important to the successful design of a "comfortable" space. Unfortunately, not all the necessary data are currently available to the designer. Natural convection in enclosures has received a great deal of attention in the literature. However, past work is generally not suitable for direct use by building services designers principally because the boundary conditions around the space to be considered are usually not well defined.

The designer of a naturally ventilated space containing sources of contamination (heat, fumes, odours, etc.) first needs to be able to determine the required area of openings between the space and the surrounding ambient air to ensure an adequate air exchange rate. The most stringent design criterion with regard to possible overheating arises under no wind

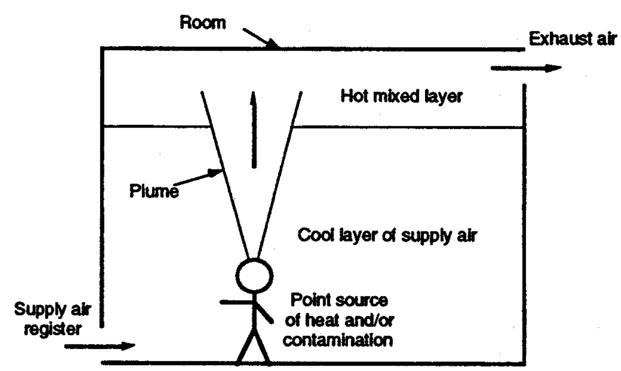

Figure 1. A displacement ventilation of a mechanically ventilated room

outside conditions. The calculations required are based on well established methods concerning the "stack effect" pressure difference (as outlined by Liddament (1986) for example) which assumes that the temperature of the air within the space is both uniform and known. However, in many situations heat sources within the space lead to a thermal (and hence contaminant) stratification within the space. This stratification may be beneficial in some regards. Displacement ventilation systems, for example, rely on the introduction of clean cool air at

lower levels to produce a stratified environment as shown in Fig. 1. This can result in greater ventilation efficiency and hence a smaller energy requirement to maintain contaminant concentration near occupants to acceptable levels as compared to a system where the air is not stratified but fully mixed (Sandberg and Blomqvist, 1989).
Plume theory may be used to predict the height of the interface between the upper hot and contaminated layer of air and the layer of cool, mechanically supplied air. This paper describes ongoing work to extend this type of analysis to naturally ventilated spaces.

A typical atrium is shown in Fig. 2 as an example of a naturally ventilated space with complex thermal and fluid flow behaviour. One may visualise the cold air from the offices surrounding the atrium as producing cool plumes descending from the balconies and the surfaces warmed by the sun as producing upward air motion that also might be regarded as plumes. In addition, the inside surface temperature of the glazing will also influence air movement and thermal comfort in the space. If the outside temperature is low a significant downdraft may occur causing low comfort levels for occupants on the floor of the atrium. Alternatively, if outside temperatures are high overheating of the upper balcony areas may occur.

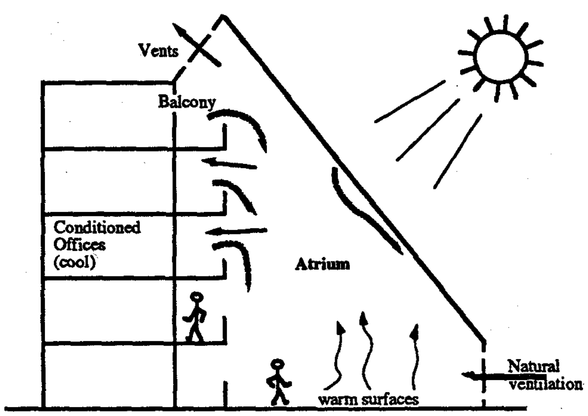

Figure 2. Multi-storey atrium

To illustrate the degree of stratification that can be found in even relatively modest atria results are presented from an extensive program of monitoring of an atrium at the University of Wollongong (Mak, 1991). The atrium is a two-storey "core" atrium (i.e. surrounded by offices). A plot of some typical data of temperature in the atrium as a function of the time of day and the height above the atrium floor is shown in Fig.3.

The thermal analysis of such spaces as atria presents the building services designer with a number of difficulties in determining: a) the *thermal stratification within the space* and hence the expected *air conditioning load on the offices adjacent* to the atrium and b) the *air conditioning load and air distribution requirements to the atrium floor level.*

One of the few options presently available to the designer seeking answers to such questions is the use of a complex Computation Fluid Dynamics (CFD) package. The disadvantages associated with this means of analysis include: a) the time and computational requirements are significant; b) few, if any, CFD packages are capable of modelling such a complex problem fully and are unlikely to have been validated against a similar full size situation; c) the boundary conditions at the walls of the enclosure are complex - they are neither constant temperature nor constant heat flux; d) the situation in reality is highly transient in nature especially since the spacial distribution of irradiated surfaces will change with the position of the sun. Thus, there is a need for a rather simpler method for estimating temperature distribution within a space to allow various design options to be explored and to be incorporated in cooling and energy analysis computer programs (Bryn, 1992).

PLUMES IN NATURALLY VENTILATED ENCLOSURES

Single heat source

Previous work by Linden et al (1990) determined that a single point source of buoyancy inside a naturally ventilated enclosure resulted in the formation of two layers of fully mixed fluid within the enclosure. Using the concept of a "point source of buoyancy" on the floor of the enclosure to represent a heat source (see Fig. 4) and the work by Morton et al (1956) on velocity profiles in plumes, Linden et al (1990) showed that the fluid in an enclosure of height H stratifies into two fully mixed layers. The lower at ambient temperature with a hot layer above an interface at z=h. The non-dimensional height, $\xi=h/H$, of the interface between the cool lower layer of air and the upper level in the enclosure is given by:

$$\frac{A^*}{H^2} = C^{1.5}\left(\frac{\xi^5}{1-\xi}\right)^{1/2} \qquad (1)$$

Where A* is the "effective" area of the top and bottom openings of the enclosure, H is the height difference between the top and bottom openings, C is an entrainment constant dependent on the rate at which the plume cross-section increases with height. A* is defined as:

$$A^* = \frac{a_t\, a_b}{\left(\frac{1}{2c}(a_t^2 + a_b^2)\right)^{1/2}} \qquad (2)$$

Where a_t, a_b and c are the top and bottom opening areas and the associated pressure loss coefficient, respectively. The lower layer being of density equal to the surrounding ambient fluid and an upper layer of density equal to that of the plume passing through the interface between the two layers. If there are n sources of equal strength present on the floor of the enclosure, then the non-dimensional height of the interface, ξ, is given by:

$$\frac{1}{n}\frac{A^*}{H^2} = C^{1.5}\left(\frac{\xi^5}{1-\xi}\right)^{1/2} \qquad (3)$$

In addition, the temperature difference ΔT_D between the two layers of air may be estimated as:

$$\Delta T_D = 24\, W^{2/3}\, h^{-5/3} \qquad (4)$$

where W is the strength of the heat source in kilowatts.

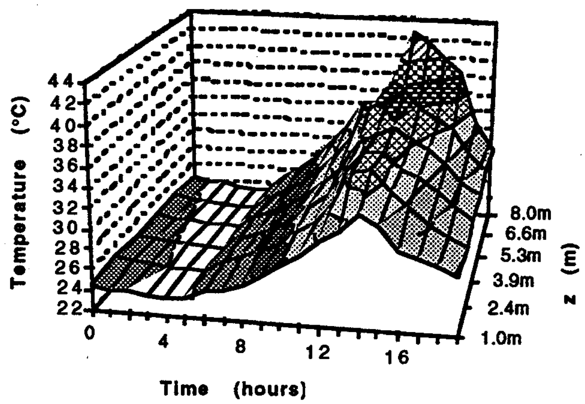

Figure 3. The Illawarra Technology Corporation Building Atrium: temperatures on 14th February 1991

These results are derived using the plume theory of Morton et al which gives the volume flow, Q, and reduced gravity, G', of fluid in a plume as a function of height, z, above a buoyancy source of strength B.

$$Q = C\,(B\,z^5)^{1/3} \qquad (5)$$

$$G' = \frac{1}{C}(B^2 z^{-5})^{1/3} \qquad (6)$$

Here $C = \frac{6}{5}\alpha(\frac{9}{10}\alpha)^{1/3}\pi^{2/3}$, α being the entrainment constant for the plume (taken as 0.082 throughout this work) and buoyancy flux B is the product of Q and G' at any cross section of the plume. Reduced gravity, G', is defined as:

$$G' = \frac{g\,(\rho - \rho_\infty)}{\rho_\infty} \qquad (7)$$

Where g is gravitational acceleration, ρ is the density of the fluid in the plume and ρ_∞ is the density of the fluid through which the plume is rising (or falling).

The approach of Linden et al (1990) has been extended recently by Linden and the present author to cover the case of two sources of buoyancy either of the same or opposite sign. The fluid mechanics are similar to the single source case but the flow analysis is complicated by the fact the stronger of the two plumes develops through a region of two distinct values of density. To analyse this situation the fluid dynamics within the stronger plume have been treated in a manner similar to that by Morton (1959) in his paper on "forced plumes", i.e. plumes evolving from sources of both finite volume and momentum flux.

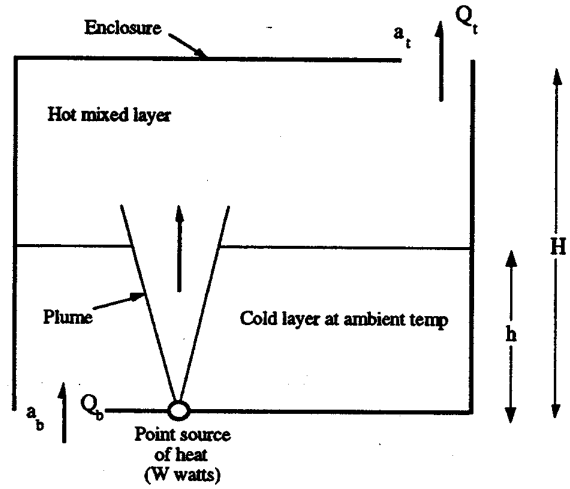

Figure 4. Ventilated enclosure with a single source of heat

Two-plumes with buoyancy of the same sign

A schematic of two point sources of positive buoyancy forming plumes in a ventilated enclosure is shown in Figure 5. Three layers of fluid of different densities are formed. Both sources (B_1 and B_2) are assumed to be "virtual" in that they each release a finite quantity of buoyancy (equivalent to heat in the thermal situation) but zero mass and momentum; i.e. they produce "unforced plumes". These plumes develop through Layer 0 of the same density as the ambient fluid. A distinct interface then occurs at a height $z = h_1$. Here the weaker plume mixes with Layer 1 and the temperature of the plume, T_{11}, is assumed to be equal to that of the fluid layer, T_1. The stronger plume from source B_2 passes through Layer 1 and mixes with Layer 2 at height $z = h_2$, where $T_{22} = T_2$. The volume flux through the top and bottom openings is determined by the stack effect resulting from the two warm layers of air, ie:

$$Q_t = Q_b = A^*(G'_{22}\,(H - h_2) + G'_{11}\,(h_2 - h_1))^{1/2} \qquad (8)$$

[Note: the first term of double subscripts refers first to the plume and the second term to the interface, e.g. Q_{21} is the volume flux in Plume 2 passing through Interface 1 between Layers 1 and 2].

The behaviour of the stronger plume in Layer 1 is influenced by the fact that it experiences a step change in the surrounding liquid density at $z = h_1$. This has been modelled by considering the plume to consist of two parts. The first, an "unforced" plume in Layer 0. The second a "distributed" plume of finite mass flux and momentum originating at $z = h_1$ and developing in an environment of uniform temperature.

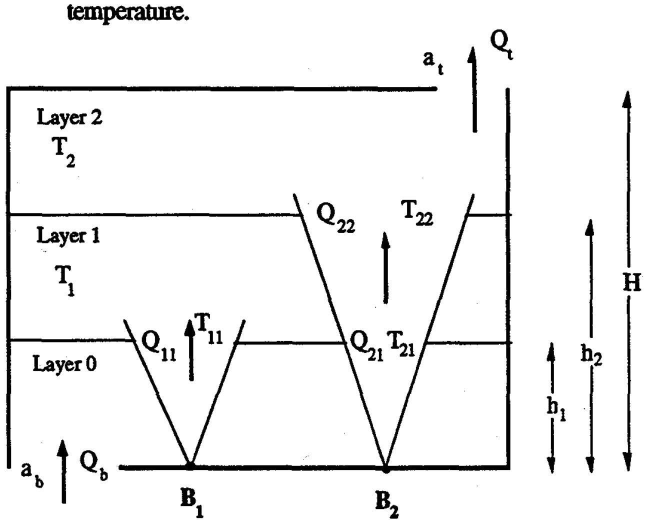

Figure 5. Ventilated enclosure with two positive buoyancy sources

It is convenient to define the ratio of the two interface heights as:

$$\frac{h_2}{h_1} = 1 + f\left(\frac{B_1}{B_2}\right) \qquad (9)$$

where $f(B_1/B_2)$ is some arbitrary function of the ratio of source strengths, B_1/B_2, to be determined. Following further calculations (Cooper and Linden, 1992) an expression equivalent to Equation (3) for the heights of the interfaces as a function of the geometry of the enclosure are given by Equation 10.

$$\frac{A^*}{H^2 C^{3/2}} = \frac{\left(1 + (B_1/B_2)^{1/3}\right)^{3/2}}{(1 + B_1/B_2)^{1/2}} \left(\frac{\xi^5}{1 - \xi - \frac{(1 - (B_1/B_2)^{2/3})}{(1 + B_1/B_2)} f\left(\frac{B_1}{B_2}\right) \xi} \right)^{1/2} \qquad (10)$$

This result is confirmed by Equation (3) in the limiting cases of a single plume (ie $B_1/B_2 = 0$) and for two plumes of equal strength (ie $B_1/B_2 = 1$). $f(B_1/B_2) = 0$ in both these cases.

Using forced plume theory it is then possible to determine f in terms of B_1/B_2 and h_1 and so close the problem. An example of the results of this analysis are shown in Fig. 6 where $\eta = h_2/H$.

As for the case of the single heat source in an enclosure the important point for designers is that the height of the air layer interfaces are independent of the strength of the two heat sources. The lowest level to which warm layers will extend is found when two sources of equal strength are present in the enclosure. As the top and bottom opening areas are increased so the interfaces will rise within the enclosure (i.e. the ventilation rate will be increased). The temperature of the two warm air layers may be calculated from the temperature of the air in the plumes entering them (i.e. T_{11} and T_{22}).

This theoretical model has been validated using water-filled scale models. The point sources of buoyancy were realised by injecting either salt solution or a mixture of methylated spirits and water into the model enclosure at the ceiling or floor. Full details will shortly be published elsewhere (Cooper and Linden, 1993).

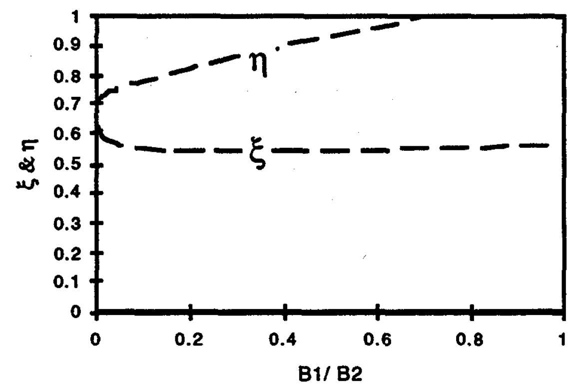

Figure 6. Non-dimensional interface heights (A^*/H^2 = 0.024)

Two-plumes with buoyancy of opposite sign

Many naturally ventilated spaces contain sources of both negatively and positively buoyant air. The

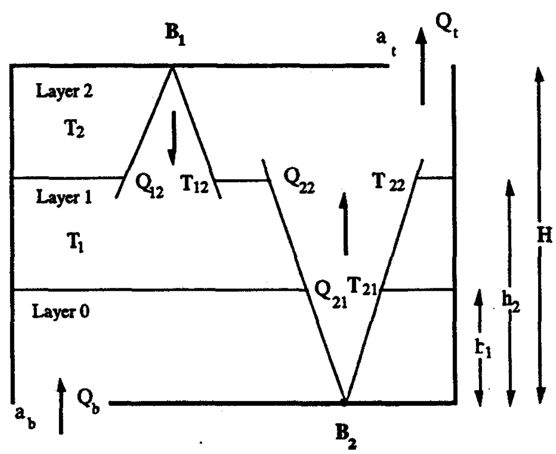

Figure 7. Ventilated enclosure with buoyancy sources of opposite sign

negatively buoyant air being generated from air conditioned rooms around the periphery of the enclosure, as in the case of the atrium shown in Fig. 2, for example. This situation has also been modelled by Cooper and Linden (1992) where the hot and cold air sources are taken to be point sources as shown in Fig. 7.

Again it is found that the heights of the two interfaces are determined simply by the geometry of the enclosure and the *ratio* of the strengths of the buoyancy sources. The absolute magnitudes of the heat sources do not theoretically affect the spatial distribution of the stratification. Interface heights determined using similar techniques to those described above and are illustrated in Fig. 8.

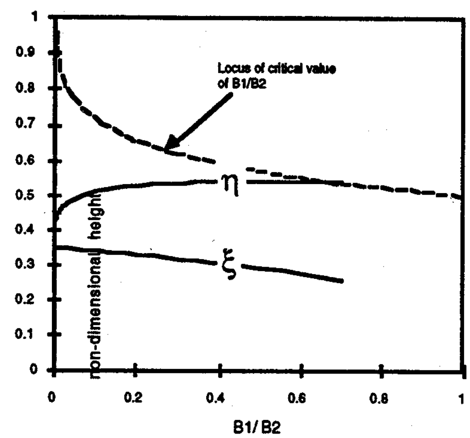

Figure 8. Theoretical prediction of interface heights for sources of opposite sign
($A^*/H^2 = 0.0029$)

An interesting phenomenon for this source geometry is that the stratification shown in Fig. 7 can only occur up to a critical value of B_1/B_2.

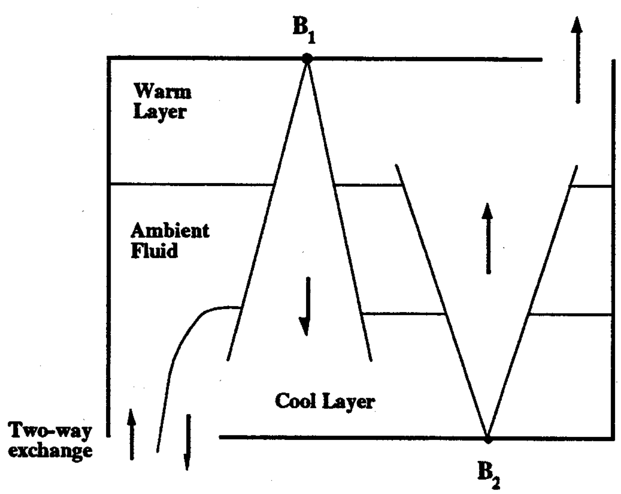

Figure 9. Buoyancy sources of opposite sign: $B_1/B2$ greater than critical value

Beyond this value the intensity of the negative source of buoyancy is sufficient to cause Layer 1 to have a density less than that of the ambient environment and a different flow regime develops. A cold layer of fluid forms at the bottom of the enclosure and there is a two way exchange of both ambient fluid and fluid from the cold layer as shown in Figure 9.

DISCUSSION

There is considerable potential for application of the type of analysis outlined above in the determination of the thermal behaviour of naturally ventilated spaces. It is unlikely that the theory as presented will model the fluid flow and thermal stratification exactly. However, it may prove extremely useful in estimating the qualitative air flow and stratification in various building design options.

The model outlined above may also be useful in visualising how a complex space may behave thermally. Take the two storey atrium mentioned above, for example. On a day such as that shown in Fig. 3, an open door from an upper level air conditioned office maintained at 24°C might exchange air with the upper level of the atrium at a mean temperature of 40°C, say. The volumetric air flow rate through a single open door may be calculated using the relation of Shaw and Whyte (1974) that gives the flow through an opening with air masses of different temperatures on either side as:

$$Q = 0.65\,\frac{W_d}{3}\left(g\,\frac{\Delta\rho}{\bar{\rho}}\right)^{1/2} H_d^{\;3/2} \qquad (11)$$

where Q is the air flow in m^3/s, $\Delta\rho$ is the density difference across the door, $\bar{\rho}$ is the mean density, g is gravitational acceleration and W_d and H_d are the width and height of the door, respectively. Thus, the open door would lead to an air exchange rate of about 350 L/s equivalent to almost 7kW of extra cooling load for the office compared to the situation with the same temperature either side of the door. Moreover, the cold (and probably contaminated) air from the door will form a plume falling into the atrium much as shown in Fig.s 2 or 9. This reduces the cooling load of the atrium and has implications for the air quality in the lower level of the atrium.

By lumping the incident solar radiation gains into one or more notional point sources on the floor of an atrium one may obtain a very crude model of the stratification that might arise in a real building. This procedure overpredicts the stratification as compared to that found in the real building on the University of Wollongong campus. The most significant factor contributing to this overprediction is the time lag between solar gain on the irradiated surfaces and the instantaneous cooling load. Modification of solar and other gains lumped into the buoyancy source of the plume by suitable cooling load factors is an option currently being investigated by the author.

A further limitation of the analysis given above in relation to building thermal design is that only point sources of heat are considered. Experimental and theoretical research is currently underway to determine the behaviour of plumes originating from hot surfaces of significant area, and the influence of such plumes on the stratification within naturally ventilated spaces.

CONCLUSIONS

The use of plume theory to model air flow and stratification within spaces with one or two point sources of heating or cooling has been demonstrated. The model predicts that one or more fully mixed layers of fluid form within the naturally ventilated space in addition to a layer of ambient air. The heights of these layers are determined by the height of the space and the size of the ventilation openings. They are not dependent on the absolute magnitude of the sources of heating or cooling.

The plume model may be used to gain qualitative insight into thermal the behaviour of large, complex, naturally ventilated spaces.

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