Stochastic Modelling and Genetic Algorithm-Based Optimal Control of Air Conditioning Systems

H.N. Lam
Department of Mechanical Engineering
University of Hong Kong

ABSTRACT

There has been widespread concern over the high energy consumption and the often less-than-satisfactory environmental control performance of most air conditioning systems relying on conventional control schemes. In this paper, a new approach to tackle the problem is presented, which aims at achieving a high quality control of the indoor thermal environment with reduced energy consumption. The rationale of the approach is to employ an optimization procedure for the control of an air conditioning system to achieve specified thermal comfort levels with minimum power input based on forecasts obtained from a stochastic mathematical model of the system.

Such a model, which embodies the interaction amongst the internal and external environments, the building structure and the air conditioning system, can be derived using time-series analysis of the actual performance data and the environmental conditions of the exterior and the conditioned space. The power consumption of the air conditioning system, indoor and outdoor air temperatures, and solar radiation are the variables included in the present time-series model.

The model is utilized to develop an optimal control strategy based on genetic algorithms. In this study, the power consumption of the air conditioner is chosen to be the objective function to be minimized using genetic algorithms against specified comfort levels based on Fanger's thermal comfort index of predicted mean vote (PMV). Computer simulation is used to compare the performance of the optimal controller using genetic algorithms and that of on-off and PI controllers. The results show that a reduction in power consumption has been achieved in most cases by the optimal controller while maintaining a higher degree of thermal comfort.

INTRODUCTION

Conventionally, air conditioning systems are often thermostatically controlled based on an indoor air set-point temperature in either simple on-off or proportional-plus-integral mode. These control algorithms completely ignore the dynamic interactions amongst the external environment, the internal environment and the air conditioning system. No use is made of the advance information regarding the primary independent variables such as solar radiation for the prediction of its effects on the indoor air temperature. The system performance is far from optimal and there are much rooms for improvement in terms of both energy consumption and thermal comfort level (Nygard-Ferguson and Scartezzini 1989). By using a non-deterministic statistical system model developed from actual performance data, a predictive control strategy can be implemented which takes into account the effects of the random disturbances present in the input variables due to their stochastic nature. The optimization procedures of the predictive control are based on genetic algorithms. Genetic algorithms are search procedures based on the law of natural selection which emulate the adaptation of a species to its environment according to genetic rules. It has

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H. N. Lam, Lecturer,
Department of Mechanical Engineering,
University of Hong Kong,
Pokfulam Road, HONG KONG.
Phone: (852)8592640 Fax: (852)8585415
Email: hremlhn@hkucc.bitnet

been shown that genetic algorithms are capable of solving a large class of global optimization problems subject to system constraints and are well-suited to computer manipulations.

TIME-SERIES MODEL OF AIR CONDITIONING SYSTEM

In the present study, a simple window-type air conditioner employing mechanical vapour compression was chosen, which served a room in the Building Services Laboratories in the University of Hong Kong. A microcomputer-based data acquisition system consisting of a front-end unit and a supervisory microcomputer was installed inside the room to monitor the operation of the air conditioner, the internal environment and the external environment. The following parameters were monitored by using electrical power meter, temperature sensors, humidity sensor and pyranometer: the power consumption of the air conditioner, the indoor air temperature and relative humidity, the outdoor air temperature and the solar radiation. A sampling period of 5 minutes was used in the data logging. A total of five days' data was obtained for analysis.

The approach to the derivation of a dynamic model of the system of the air conditioner in interaction with the internal and external environments is based on the methodology of AutoRegressive Moving Average (ARMA) modelling (Franklin and Powell 1980). In this modelling procedure, the system variables are classified into three categories: output variables, controlled input variables and disturbance variables. The indoor air temperature was taken as the output, the power consumption of the air conditioner as the controlled input, and the solar radiation, outdoor air temperature and indoor air relative humidity as the disturbances. The model is of the form given in Equation (1). The form of the model can be modified by applying steady-state rationalization as suggested by Crawford (Crawford and Shirey 1987), in order to reduce the number of independent coefficients. The resultant form of the model is as shown in Equation (2).

Time-series of the variables involved were used to determine the coefficients in the model by means of multiple linear regression analysis (Montgomery 1992). A stepwise regression procedure was employed to determine which past time steps for each variable were significant to be included in the final model. A statistical analysis computer package running on a microcomputer was used to perform the numerical analysis. The data for the first day were used for initialization of the model and the remaining data were available for model development. The final model obtained for the indoor temperature, which was based on a four-day development period, can be expressed by Equation (3). There is no indoor air relative humidity term appearing in the final model, which indicates that this variable has no significant effects on the system performance.

The accuracy of the model can be determined by calculating the root-mean-square error between the predicted and actual indoor air temperature over any one day within the four-day period. The results show that the average error over the four-day period is only 0.0899 °C with a maximum of 0.095 °C for the fourth day and a minimum of 0.079 °C for the second day. The accuracy of the model in predicting the indoor air temperature for a specified period outside the model development period was estimated by using models developed based on data for half-day, one day, two day and three days to predict the indoor air temperature for the subsequent day. The results indicate that, with the exception of the 'half-day model', the accuracy of prediction is very steady, with an average root-mean-square error of about 0.11 °C. It can therefore be deduced that the 'four-day model' can be used with confidence for subsequent one-day predictions.

THERMAL COMFORT CRITERION

According to the theory of Fanger (Fanger 1972), the conditions for optimal thermal comfort for a person in a given environment are expressed by the Fanger's comfort equation. For any given type of clothing worn and activity engaged in, it is possible to use the comfort equation to determine all reasonable combinations of air temperature, air humidity, mean radiant temperature and relative air velocity which will create optimal thermal comfort for persons under steady state conditions. Based on the comfort equation, Fanger developed a thermal sensation index called Predicted Mean Vote (PMV) which can be used to predict the thermal sensation of a person for any given combination of the six parameters mentioned above when the comfort equation is not satisfied.

In this study, the PMV value was used as a comfort variable for the control of the air conditioner to achieve specified comfort levels. In the calculation of PMV, the following assumptions were made. The mean radiant temperature, t_{mrt} was taken to be the same as the indoor air temperatue. The activity level, M/A_{Du} was set to 100 kcal hr^{-1} m^{-2}. The clothing insulation, I_{cl} , was set to 1 clo. The clothing factor, f_{cl}, was set to 1.15. The mechanical efficiency was set to 0.05. An average air relative humidity of 40% was used.

OPTIMIZATION USING GENETIC ALGORITHMS

Genetic algorithms are search procedures based on the law of natural selection which emulate the adaptation of a species to its environment according to genetic rules. Genetic algorithms have been shown to be capable of solving a large class of global optimization problems and well-suited to computer manipulations (Goldberg 1989). In this optimization method, the parameter set is usually coded into a finite-length binary string which is used for evaluating the objective function. For constrained optimization problems, a penalty function can be included in the objective function evaluation to allow for the cost associated with a constraint violation. Probabilistic rules are used to search from a population of points and to determine the string which gives the optimized objective function. In this study, three simple genetic operators were used: Reproduction, Crossover and Mutation. Reproduction is a process in which individual strings are copied according to their objective function (or fitness function) values. Strings which have a higher fitness value have higher probability of having one or more offspring in the next generation. In Crossover, newly reproduced strings are first mated at random and then a random number of individual bits in a mated pair of strings are swapped to yield a pair of new strings. Mutation is the occasional random alteration of the value of a string position within a string.

In this study, the power consumption of the air conditioner was taken as the objective function to be minimized subject to the constraints that the required range of PMV values should be satisfied. To transform this minimization problem to a maximization problem so that it can be solved by using genetic algorithms, the following form for the raw fitness was used:

$$RawFitness = MaxObjective - Objective \qquad (4)$$

where *MaxObjective* is the maximum power consumption of air conditioner in the current generation and *Objective* is the power consumption of air conditioner for the string under test.

The raw fitness was scaled using a linear scaling of the following form:

$$ScaledFitness = (a)\,(RawFitness) + (b)\,(PenaltyFunction) \qquad (5)$$

where *a* and *b* are suitable weighting factors.

For the case of the PMV index lying within the differential range, the following relationships were used:

$a = 10^6$
$b = 10^4$

$$PenaltyFunction = 1/(|PMV - SetPMV| + AirVelocity) \qquad (6)$$

Otherwise, the following relationships were used:

$a = 10^{-2}$
$b = 10$

$$Penalty\ Function = 1/[\exp(3\ \ln(|PMV - SetPMV|) + Airvelocity))] \qquad (7)$$

The PMV index was determined using the appropriate equations (Fanger 1972) based on the indoor air temperature predicted by the time-series model described by Equation (3). The genetic algorithm was coded in a microcomputer version of Pascal employing the following parameter settings:

Population size: 20
Number of parameters: 2 (power consumption and air velocity)
Number of bits for parameters 1 & 2: 6 & 4
Length of string: 10
Range of parameter 1: 0.45 to 2.50 (kW)
Range of parameter 2: 0.62 to 0.63 (m s^{-1})
Set PMV value: -0.14
PMV differential: 0.04
Probability of crossover: 0.95 (one-point crossover)
Probability of mutation: 0.01
Maximum number of generation: 100
Optimum objective tolerance: 0.00001

The genetic algorithm program was used successfully to determine the optimal power consumption of the air conditioner for each time step of five minutes in order to comply closely with the specified PMV range. Running on a 486-microcomputer at 33MHz, the program took, on average, 35 seconds to complete the optimization with less than 20 generations for one time step.

GENETIC ALGORITHM CONTROL VERSUS CONVENTIONAL CONTROL SCHEMES

In order to study the performance of the optimal controller implemented using time-series modelling and genetic algorithms, a comparison by computer simulation was made between this control scheme and two conventional control methods, viz. on-off control and proportional plus integral control.

The three cases chosen for direct comparison were:
(a) On-off control with a set-point at 17.9 °C and a differential of 1.6 °C.
(b) PI control with a set-point at 17.9 °C and a proportional constant of 0.10.
(c) GA control with a PMV set-point at -0.14 and a differential of 0.04.

The indoor air temperature, PMV index, power consumption and energy consumption of the air conditioner for each of these cases for a period of 11 hours were simulated and compared. The results were plotted in Figures 1 to 4. Fig. 1 shows the indoor air temperature curves. It can be noticed that the curve for GA control is very steady. By comparison, the GA control curve in Fig. 2 of indoor air PMV value is also the most steady one. Fig. 3 shows the power consumption curves. Here it can be seen that the power consumption under GA control varies swiftly in response to the needs as predicted by the time-series model. Fig. 4 is a vertical bar chart showing the total energy consumptions of the air conditioner for the three cases. Two comparisons were made; one for the entire 11-hour period and the other for the 4-hour period following the initial 6-hour period for the initialization of the temperature prediction model. For the former case, the energy consumption for GA control is about 2.6% higher and 5.3% lower when compared with on-off and PI controls respectively. For the latter case, the corresponding savings in energy consumption are about 21.4% and 17.8%. It must, however, be pointed out that the comparisons on power and energy consumptions for the three control modes were not made under identical conditions although the conditions were very similar. The main difficulty lies in the prediction of the characteristics of the GA controller corresponding to a set PMV value and differential such that all three controllers will give the same mean indoor air temperature.

CONCLUSIONS

In this research project, it was found that a stochastic dynamic model of an air conditioning system can be derived statistically from the actual system performance data. The model can be used to make reasonably accurate short-term predictions of the variation of the indoor air temperature with the power consumption of the air conditioning system and the outdoor climate. An optimal control strategy based on genetic algorithms was developed and implemented for the air conditioning system by utilizing the dynamic model derived. Computer simulation studies of the performance of the optimal controller in comparison with two conventional control schemes were made, which showed that the GA controller was superior to the two conventional control schemes when control quality and energy consumption were taken into consideration.

DISCUSSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The findings of the research are encouraging since they indicate that advanced computer control can be designed for and applied to an air conditioning system based on its operating data. These data are easily available from computerized energy management and control systems which are becoming increasingly popular for buildings. If on-line model identification can be achieved, it will mean that the model can be updated periodically with the new incoming data to yield more accurate predictions for use by the genetic algorithm controller. The modelling procedures employed in this study can only be carried out off-line and are very time consuming. Therefore, alternative methods of model development should be investigated. Currently, an automatic state-space model identification procedure is being studied which can deal with multivariate cases efficiently. The present work is only concerned with the computer simulation study of the performance of the optimal controller in comparison with other conventional controllers. Implementation of the optimal control scheme should be carried out on an actual air conditioning system to validate the simulated performance. A split-unit air conditioner with a three-phase compressor has been chosen for study and work on the control hardware and software is now underway. More research effort should also be made to investigate ways for improving the efficiency and robustness of genetic algorithms such as establishing better guidelines for constructing fitness function and quantifying the advantages of advanced operators.

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EQUATIONS

$$
\begin{aligned}
T_a(k) = {} & \sum_{i=1}^{n_a} a_i T_a(k-i) + \sum_{i=0}^{n_b} b_i T_o(k-i) + \sum_{i=0}^{n_c} c_i Q_s(k-i) \\
& + \sum_{i=0}^{n_d} d_i \phi_a(k-i) + \sum_{i=0}^{n_e} e_i Q_p(k-i) + \sum_{i=0}^{n_f} f_i T_o(k-i) Q_p(k-i) + g
\end{aligned}
\tag{1}
$$

where $T_a(k)$ = indoor air temperature at time step k, °C

$\phi_a(k)$ = indoor air relative humidity at time step k, %

$T_o(k)$ = outdoor air temperature at time step k, °C

$Q_s(k)$ = horizontal solar radiation at time step k, $\mathrm{kWm^{-2}}$

$Q_p(k)$ = power consumption of air conditioner at time step k, kW

g = model constant

n_a - n_f = number of past time steps for each variable

a_i - f_i = system coefficients for each variable at each past time step

$$
\begin{aligned}
T_a(k) - T_o(k) = {} & \sum_{i=1}^{n_a} a_i [T_a(k-i) - T_o(k)] + \sum_{i=1}^{n_b} b_i [T_o(k-i) - T_o(k)] \\
& + \sum_{i=0}^{n_c} c_i Q_s(k-i) + \sum_{i=0}^{n_d} d_i \phi_a(k-i) + \sum_{i=0}^{n_e} e_i Q_p(k-i) \\
& + \sum_{i=0}^{n_f} f_i T_o(k-i) Q_p(k-i) + g
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
T_a(k) = {} & 0.83945 T_a(k-1) + 0.0332824 Q_p(k-23) \\
& + 0.05600 T_a(k-21) + 0.0110924 Q_p(k-30) \\
& + 0.04475 T_a(k-55) + 0.0190783 Q_p(k-57) \\
& + 0.03808 T_a(k-59) + 0.0302642 Q_p(k-61) \\
& - 0.0091326 T_o(k) Q_p(k) + 0.02172 T_o(k) \\
& - 0.0073647 T_o(k-1) Q_p(k-1) - 0.140226 Q_s(k-46) \\
& + 0.0040325 T_o(k-2) Q_p(k-2) + 0.14103 \\
& + 0.0019223 T_o(k-3) Q_p(k-3)
\end{aligned}
\tag{3}
$$

FIGURES

Fig.1: Indoor Air Temperature

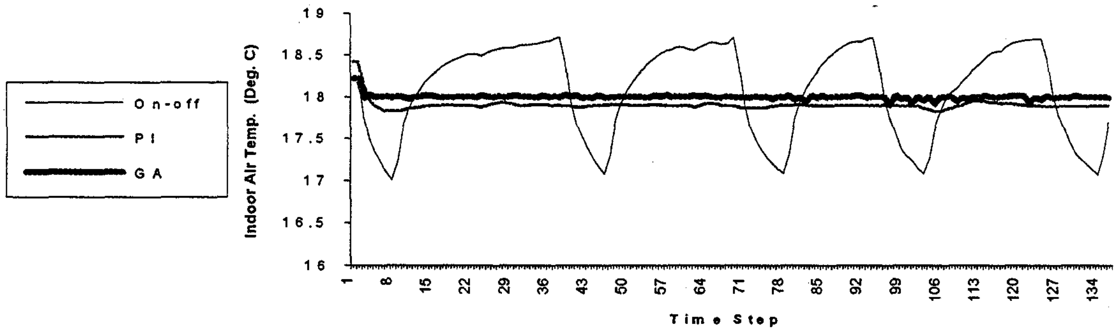

Fig.2: Indoor Air PMV Value

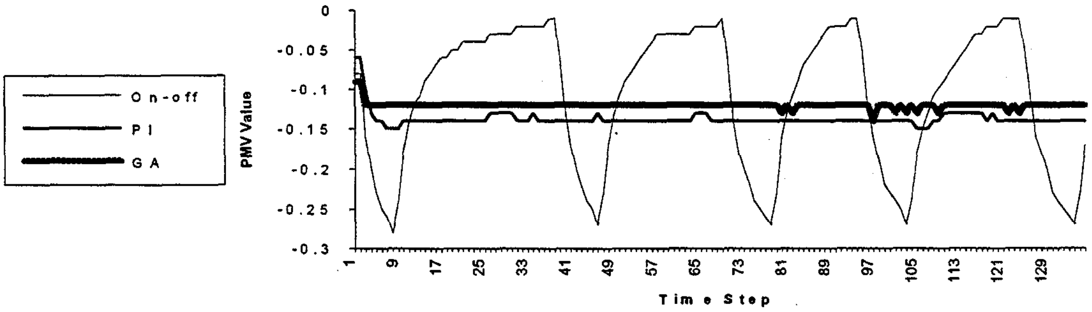

Fig.3: Power Consumption

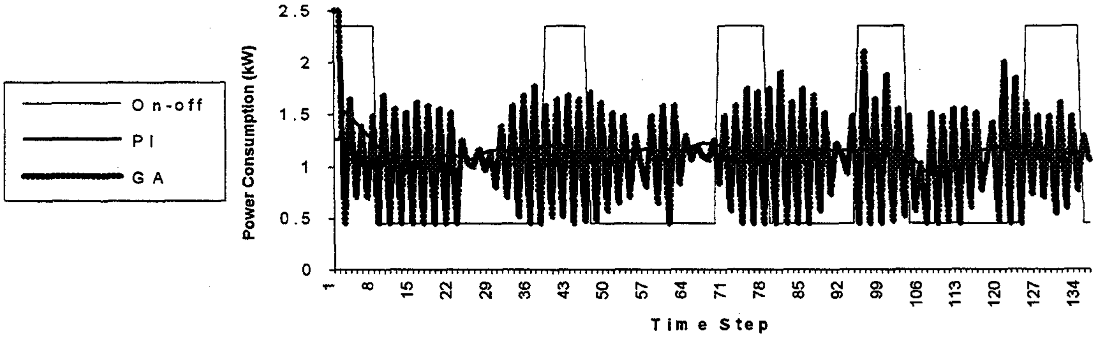

Fig.4: Energy Consumption