A new technique for thermal modelling of building : THE MODAL SYNTHESIS

B.Flament, I.Blanc Sommereux, A.Neveu
GISE (Groupe Informatique et Systèmes Energétiques)
EMP / ENPC La Courtine F-93167 Noisy le Grand Cedex
membre du GER ALMETH France

Abstract

This paper presents a new technique for the thermal systems modelling. This method, based on the structural modularity of a thermal system such as a building belongs to the modal methods family. It especially allows to obtain accurate enough reduced thermal models of large system. In this paper, the main advantages of this method are specified (recurrent, ...). Its funtamentals are given and the results are illustrated by two examples (three layers wall and bizone building).

Nomenclature :

matrix and vectors :

B : matrix of the driving force.

F : matrix of the eigenvalues.

G : matrix of the direct effect.

H : observation matrix.

P : matrix of the eigenvectors.

S : matrix of the static field.

U : vector of the inputs.

X : state vector.

Y : vector of the outputs.

index :

k : index of a local modal model n° k .

c : index related to the input on the linked up frontier.

r : index related to the outputs on the linked up frontier.

A : index related to the assembled model.

1 From modal analysis to modal synthesis

The main interest of the modal analysis technique for coupled heat transfer modelling (Lefebvre and al 1990), is to represent the thermal dynamic behaviour of a system through its eigenelements (eigenmodes and eigenvalues). The latter which are intrinsic characteristics of the thermal system, constitute a basis called modal basis. A truncation of this basis which allows to obtain *a reduced modal model*, is very efficient, as only a few eigenelements are necessary to keep the model accurate enough.

Thus, a modal model is a mathematical representation of a thermal system in an eigenbasis. In order to obtain such a model, a transformation of the physical basis has to be performed. This transformation is linked with the resolution of the eigenvalue problem corresponding to the spatial heat operator which determines the thermal system behaviour (Khoury and Neveu 1989).

This technique has shown its various possibilities, either for identification methods (Neirac 1989) , or for the simplified development tools (Peuportier and Blanc Sommereux 1990). Nevertheless, the use of this method to study complex systems leads to compute eigenelements of large matrix (computation time, accuracy...). To avoid this kind of problem, some "substructuration methods" have been developed in mechanical engineering since 1960 (Imbert 1984) including *modal synthesis* methods (Bourquin 1989). Thus starting from these different concepts we developed a new formulation of the *modal synthesis* appropriate for building thermal modelling.

The aim of this paper is to present this new method in thermal engineering which is now developed in the group GISE. This technique is based on the possible modular description of a complex thermal system such as a building. This approach is compatible with a modular data structure that allows to take advantage of advanced software engineering techniques such as object oriented programming, and opens the field to a complete modular structure (starting from the physical description to its underlaying mathematical representation).

We present first the fundamentals of the method then, two examples (a multilayer wall and a multizone building) are given to illustrate its practical interest.

2 The modal synthesis : its fundamentals

A complex thermal system can be described as the coupling of various components. Each component associated with local boundary conditions can be studied with the modal analysis method. This first step allows to obtain a reduced modal model for each component : ***a local modal model.*** Then these reduced models are linked up to obtain a model of the whole system : ***the model of synthesis.*** The method briefly described above corresponds to the modal synthesis technique. Its analytical formalism has been established (Gise 1990) taking into account local models which can be described by non selfadjoint heat operator, but assuming linearity and stationarity. Its main results are now given.

First, let us precise some concepts and assumptions about the local models. The local models that we consider are related to components which are issued from an arbitrary cutting out of a more complex system. But in practice, each model of component is defined ignoring the complex system in which it will belong. (Thus, libraries of modal models can be created). That is why we assume that these models are delimitated by frontiers being able to be linked up with another model's frontier. On each frontier, some driving forces are applied. They are unknown and computed during the linkage phasis.

For a considered coupling level only one frontier per model is linked up. Then we distinguish the driving forces applied on this frontier (U_{kc}) and the others (U_k) which are determined during other coupling levels.

We also assume a uniform temperature field , and a uniform heat flow for each frontier.

In these conditions, each local k modal model can be represented by the following equations :

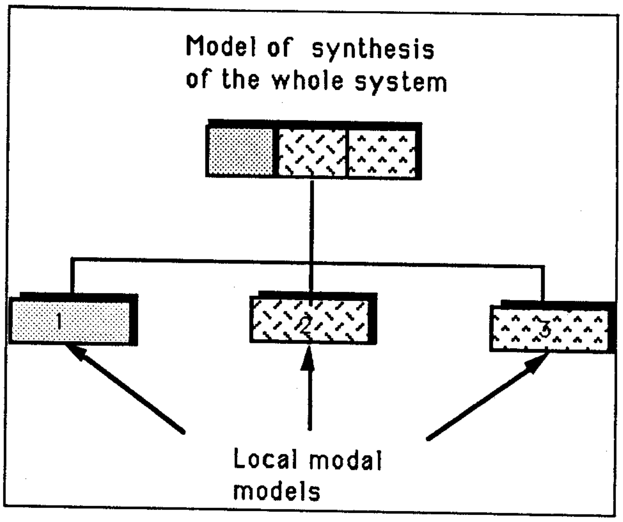

Figure 1: Case of a straight coupling

- state equation :

$$\dot{X}_k = [F_k]X_k + [B_{kc}]\dot{U}_{kc} + [B_k]\dot{U}_k \quad (1)$$

- linked up frontier outputs

$$Y_{kr} = [H_{kr}]X_k + [G_{krc}]U_{kc} + [G_{kr}]U_k \quad (2)$$

- other outputs

$$Y_k = [H_k]X_k + [G_{kc}]U_{kc} + [G_k]U_k \quad (3)$$

- temperature field

$$T_k = [P_k]X_k + [S_{kc}]U_{kc} + [S_k]U_k \quad (4)$$

where U_{kc} represents the unknown driving force applied on the frontier linked up.

To build a model of synthesis several possibilities can be considered :

1. The local modal models can be linked up straight ; in this case, the model of synthesis is built with only one coupling level (figure 1).
2. The model of synthesis can be also built by linking up at once two local modal models . In this case several coupling levels are needed (figure 2).
3. We can also link up at once several models, with several coupling levels.

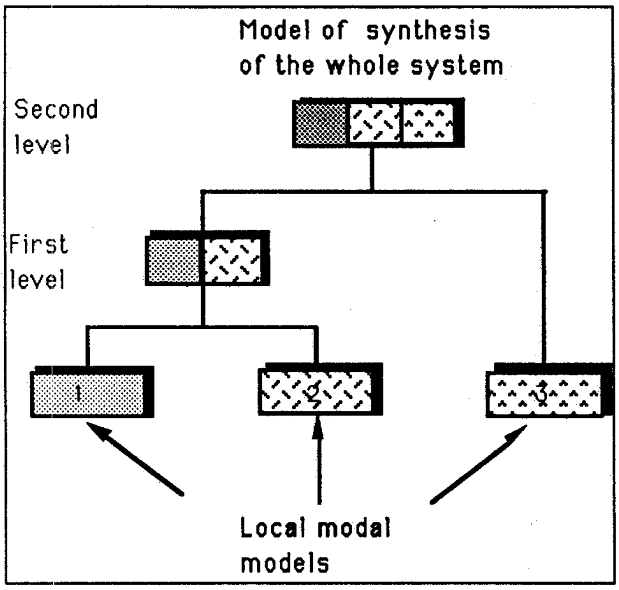

Figure 2: several coupling levels

The second method is more modular and more systematic. That is the one which is presented in the following.

Let us now describe the three main steps which allow to obtain a model of synthesis :

- The first step consists in **gathering** the equations related to the **local modal model**, in order to obtain a gathered model :

$$\begin{aligned} \dot{X} &= [F]X + [B_c]\dot{U}_c + [B]\dot{U} \\ Y &= [H]X + [G_c]U_c + [G]U \end{aligned} \qquad (5)$$

- The second step is called **"linkage phasis"** : to constitute the complete system with its components, a model of linking up between these components is needed. This latter is described by physical conditions (thermal contact, radiation...) and by geometrical conditions :
  - either coupling frontiers and natural ones are intermingled : edge to edge coupling (figure 3).
  - or the models are overlapping each over : coupling with a common area (figure 4).

  One can notice that these conditions do not have to be confused with the local boundary conditions applied to the frontiers of the local models.

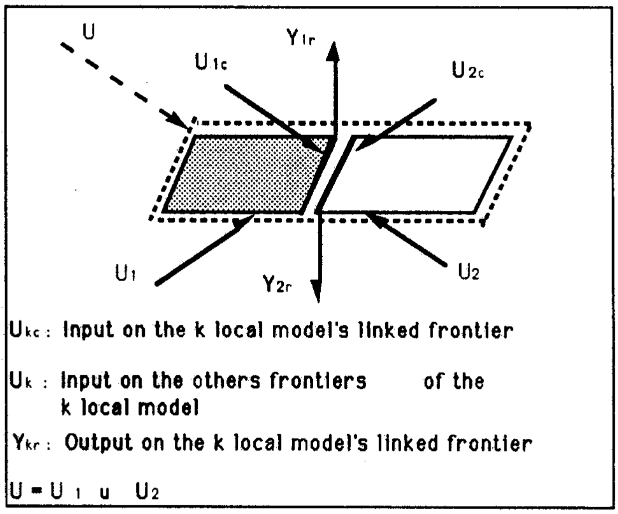

Figure 3: Edge to edge coupling

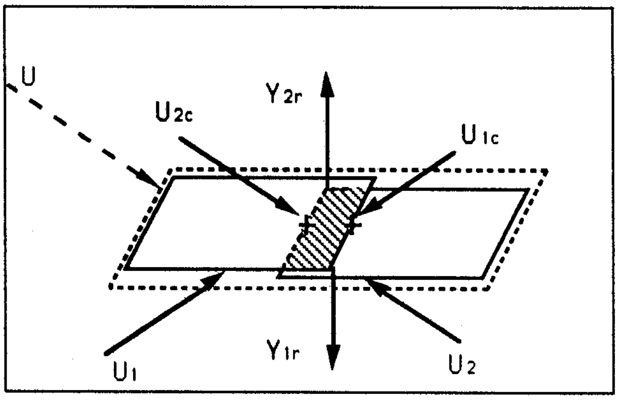

Figure 4: coupling with a common area

These considerations about linkage models are not specific to the modal synthesis but can be used for the development of a general theory for connecting models.

This second step allows to link the unknown driving forces (U_c) to the local model state (X) and to the other driving forces (U), in order to obtain the following equation of connection .

$$U_c = f(X,U) \qquad (6)$$

- The third step consists in the **assembling** of the local models gathered equations (5), with the equation of connexion (6). The result of this assembling gives **the model of synthesis** as following :

$$\begin{array}{lcl} [N]\dot{X} & = & [F]X + [B_A]\dot{U} \\ Y & = & [H_A]X + [G_A]U \end{array} \quad (7)$$

We can notice that, though this model is issued from modal models, it is not a modal type, because of the [N] matrix.

But we have shown (Gise 1990), that if the local models are not reduced, the model of synthesis has the same spectra of eigenvalues, as the modal model of the system obtained straight by the modal analysis (that we can call global model). Thus, when the local modal models are reduced, we obtain an approximate eigenvalues spectra.

We have also shown that it is possible to reconstitute the eigenfunctions (or the eigenvectors) of the thermal system with the eigenfunctions (or the eigenvectors) of its components modal models, with their static field, and with the eigenvectors of the model of synthesis (eigenvectors of $N^{-1}F$).

These results allow us to put the model of synthesis in a modal form as following :

$$\begin{array}{lcl} \dot{\mathcal{X}} & = & [\mathcal{F}]\mathcal{X} + [\mathcal{B}]\dot{U} \\ Y & = & [\mathcal{H}]\mathcal{X} + [\mathcal{G}]U \\ T & = & [\mathcal{P}]\mathcal{X} + [\mathcal{S}]U \end{array} \quad (8)$$

This new modal model can be reduced and then coupled with another one. Thus it is possible to repeat several time this procedure of coupling two models at once. Such method is therefore powerful as it is a recurrent method. Two applications are now given for the modal synthesis, each for a different coupling mode:

- A multilayer wall with an edge to edge coupling
- A bizone building with an overlapping of its zones

3 Case of a multilayer wall

To illustrate the method with several levels of coupling, let us consider a three layers wall made of 20 cm of concrete, 8 cm of insulation material and 5 cm of plaster, for which a model is built assuming a prescribed surface temperature on each side of the wall.

Before applying the synthesis method, each layer has to be represented by a modal model. Thus, local boundary conditions are applied on each linked frontier.

According to the kind of local boundary condition (prescribed temperature, prescribed heat flow, convection), the reduced model size will be different (number of eigenelements kept) to obtain the same accuracy. These considerations can be illustrated by a modal response spectrum (figure (5)).

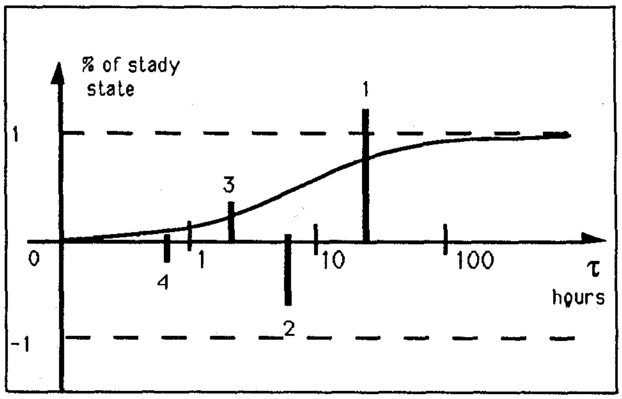

Figure 5: Modal response spectrum

On this chart, the curve response of an output (heat flow, temperature), to a step of an input (temperature, heat flow) is drawn. The eigenmodes excitation to this input can also be drawn ; they are represented by vertical lines (rays) which height is proportional to the eigenmode excitation importance, related to the input/output set considered. Thus, this chart is very useful to select the eigenmodes that have to be kept, in order to obtain a reduced modal model accurate enough.

Assuming prescribed surface temperature on one face of each layer, if the same boundary condition is applied on the other face, each eigenmode will have the same "height for a ray", on the other hand, if a heat flow is prescribed on these faces, only a few eigenmodes are dominant. That is why each local basis has been built with a prescribed surface temperature on one frontier and a prescribed heat flow on the over one.

Let us build, now, the model of synthesis of the three layers wall as it is shown on figure 6

First, the concrete layer and the insulation layer are coupled. The local modal model of these two components are reduced to their four first eigenmodes. The linking up of these two reduced modal model gives a first model of synthesis, which the response spectrum is given figure 7 (output : heat flow on the right surface of the insulation layer, input : temperature on the left surface of the concrete layer).

This chart can be compared to the same response spectrum related to a complete modal model of the two layers wall obtained straight from the modal analysis method (figure(8)).

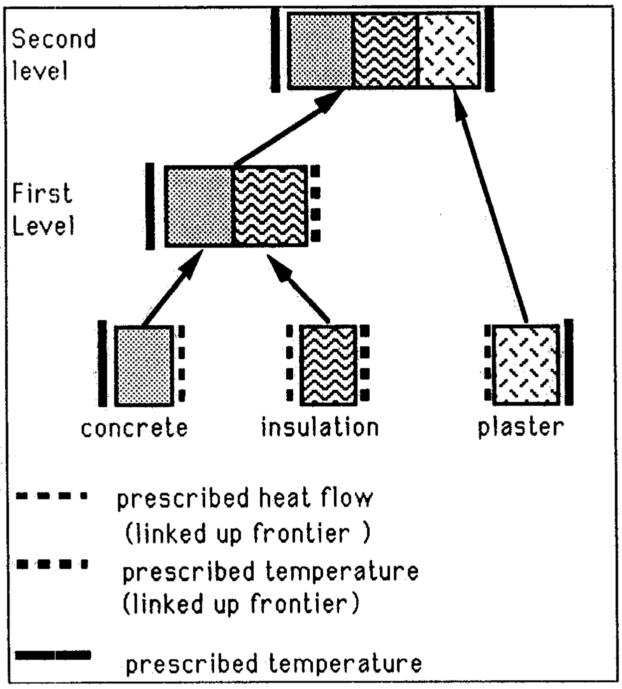

Figure 6: Model of synthesis construction of a three layers wall

This comparison shows that the eight eigenmodes of the model of synthesis correspond to the eight first main eigenmodes of the global modal model.

For the second level of coupling, the first model of synthesis is reduced to five eigenmodes (1,2,3,4,5) and linked up with the modal model of the plaster layer reduced to its first eigenmode, in order to obtain the model of synthesis of the three layers wall. Its spectrum response is given on figure 9 (output : heat flow on the right surface of the plaster layer, input :temperature on the left surface of the concrete layer).

This chart can be also compared with the same response spectrum of the three layers wall modal model obtained directly from the modal analysis method (global modal model) (figure (10)).

The curve response obtained with the final model of synthesis is very accurate. What shows the interest of the modal synthesis method. A same accuracy is obtained with the eigenmodes themselves, they fit well with the eigenmodes of the global modal model. This result is specified in the following example related to a bizone building.

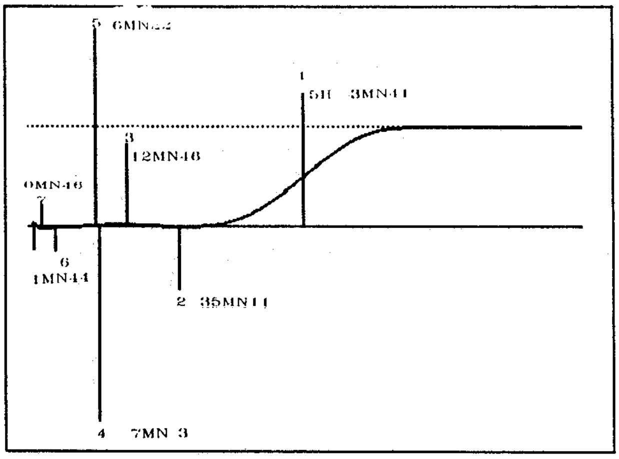

Figure 7: Response spectrum of the first model of synthesis (8 eigenmodes)

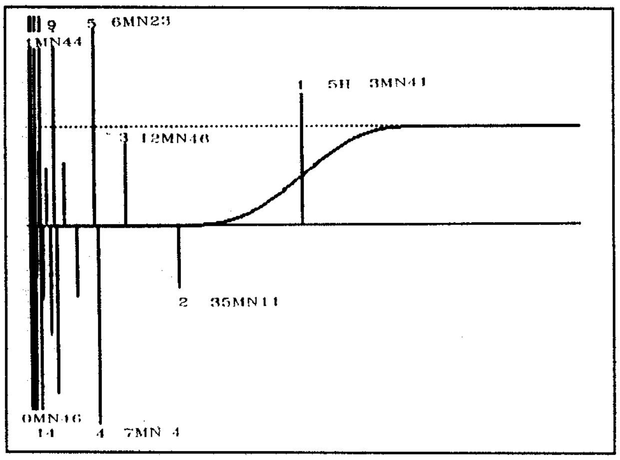

Figure 8: Response spectrum of two layers modal model (45 eigenmodes)

4 Case of a bizone building

The overlapping type coupling is well adapted for the coupling of zones in a building. As a matter of fact, the design of a multizone building lies on the assembling of zones. These zones may have been defined previously by models describing totally the zone including possible common walls. The overlapping type coupling allows to use the existing zone models without modifying them and to couple them considering common walls as overlapping zones. The modal synthesis is particularly adapted to handle such assembling of zones (the local models) in order to recreate the whole building (the global model). Moreover the modal synthesis allows to analyse the contribution of each zone through its eigenmodes to the ther-

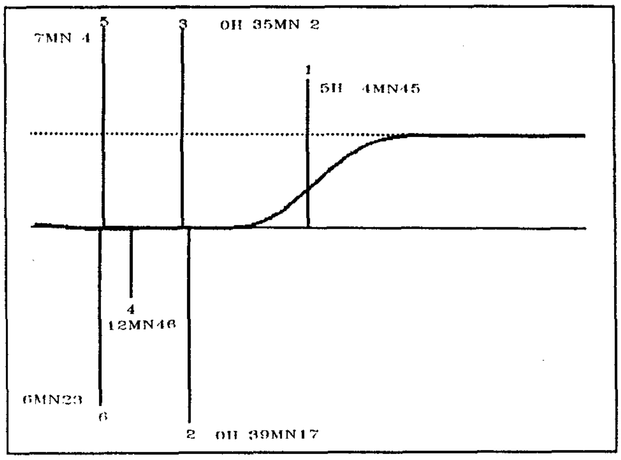

Figure 9: Response spectrum of the model of synthesis of the three layers wall (6 eigenmodes)

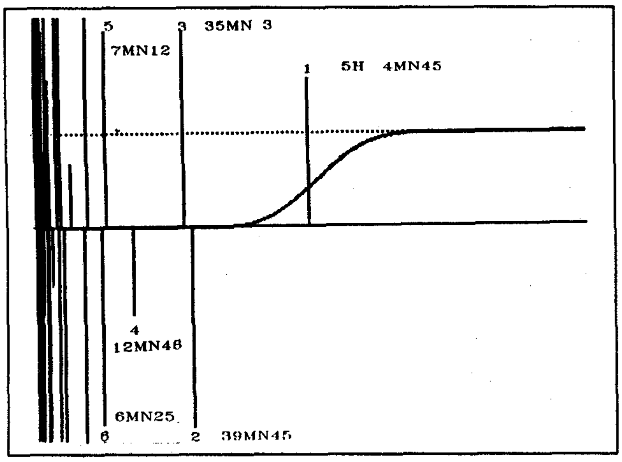

Figure 10: Response spectrum of the three layers wall complete modal model (60 eigenmodes)

mal behaviour of the whole without using any simulation. The eigenmodes of a building are spatial thermograms. Their visual study is immediate and helps to the understanding of the modal synthesis.

An exemple of a bizone building is now treated (see Figure 11)

The eigenmodes of each zone constituting the building are given on figure 12 and figure 13. These modal models were generated with "COMFIE" (Peuportier and Blanc Sommereux 1990).

A physical meaning of some of the eigenmodes can be given: for example the eigenmode associated with the highest time constant is an approximation of the steady state. It usually has a dominant effect and as a consequence, (Lefebvre 1989), proposed a simplified computation of this first time constant as a pertinent parameter for the inertia of a building. When considering the first time constant of both zones, we can notice that zone 1 has a higher inertia than zone 2. This is explained by the fact that the thickness of the floor in zone 1 is twice as the thickness of the floor for zone 2. The wall thermograms are well identified for the first eigenmode for both zones: wall n° 1 is the common wall, walls n° 2,3,4,7,8 and 11 have a similar spatial representation as they have the same composition, walls n° 4 and 9 are the ceilings, walls n° 6 and 10 are the floors. The ray on the left is the air response for each eigenmode. When an eigenmode has no influence on the air, no ray is visible (this is the case for eigenmodes number 2 and 3). The modal synthesis is now applied to these modal models for both zones using the overlapping coupling technique. The eigenmodes issued from the modal synthesis are given on figure 14 .

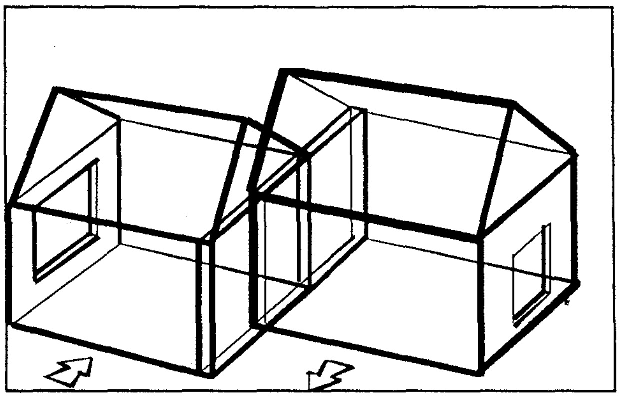

Figure 11: coupling with a common area for a bizone building

Having coupled both zones, the eigenmodes are now represented through all walls. The common wall (n° 1) is duplicated on Figure 14 as the overlapping coupling handles it twice (the shapes are symetrical as it depends on which zone the common wall belong to). The respective importance of each zone can be deduced from the height of the air rays and this helps to the understanding on how the synthesis eigenmodes are constituted. By studying the shape of the thermograms for the first synthesis eigenmode, we deduce that it is made, for one part, of the first eigenmode from zone 1, and for another part, of the first eigenmode from zone 2. The eigenvalue of this first eigenmode (61H 40MN) is higher than the first eigenvalues of each zone (59H 43MN and 19H 37MN).

A comparison can be performed now with the

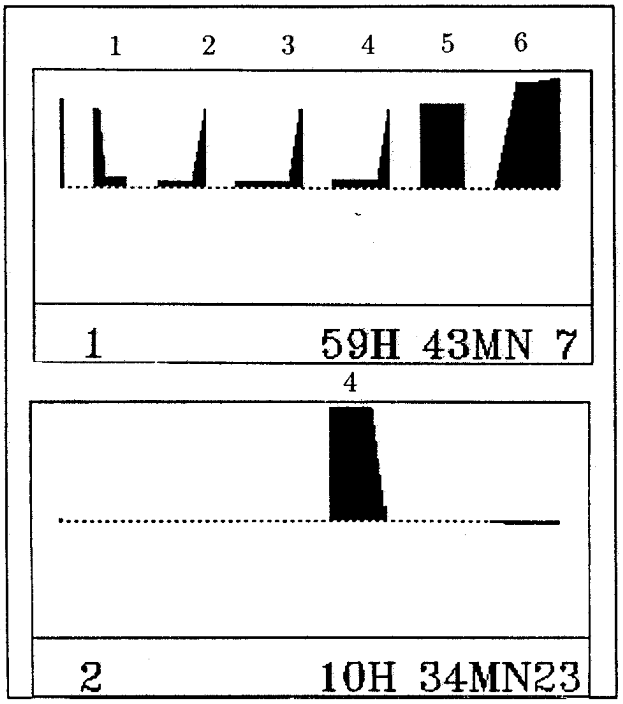

Figure 12: eigenmodes for zone 1

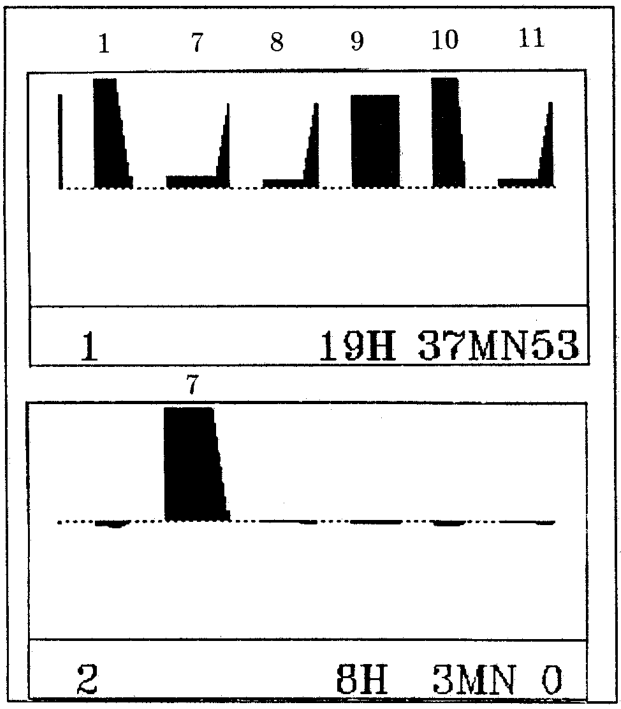

Figure 13: eigenmodes for zone 2

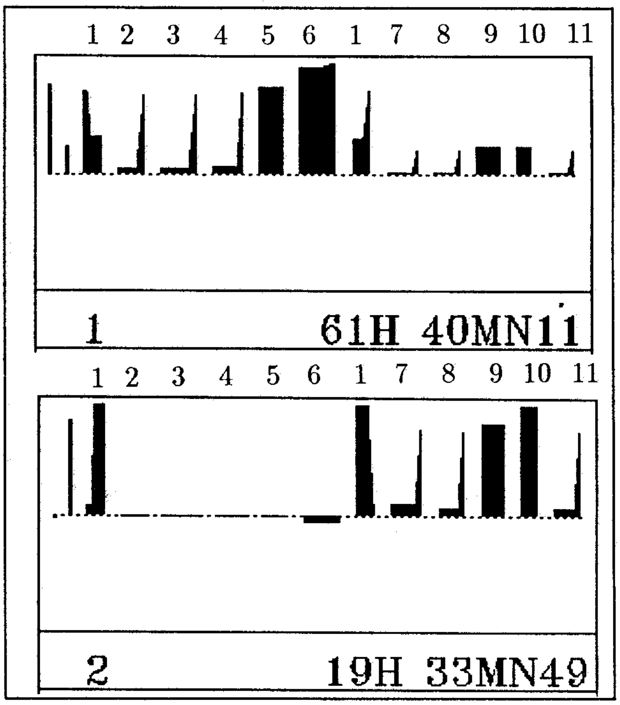

Figure 14: eigenmodes for the model of synthesis

modal model of the whole building (without any synthesis step). Such global model has been generated with MINERVE (Neveu and al 1987). The corresponding eigenmodes are visualized in Figure 15.

Values for time constants are close and the shape of the thermograms are identical. Differences for the thermograms come from the duplicate representation of the common wall (*n°* 1) in the synthesis model (Figure 14) and from additional representation for the windows for MINERVE (Figure 15,walls n° 12 and 13). Differences for the eigenvalues comes from the discretization of both models (MINERVE and COMFIE). The node grid in Minerve was set to one node per layer while for COMFIE layers where combined (layers having a low inertia such as insulation layer) and therefore the node grids were not exactly identical. Besides these minor differences, the accordance between the two models is good and show how well performs the modal synthesis.

The modal synthesis is very powerful for the design of multizone buildings as it only requires a separate model for each zone (possibly coming from an already existing library) in order to reconstitute the whole building. As a consequence complex problems can be splitted in simpler ones and local behaviour of particular zones or components can be identified as a contribution to the whole behaviour.

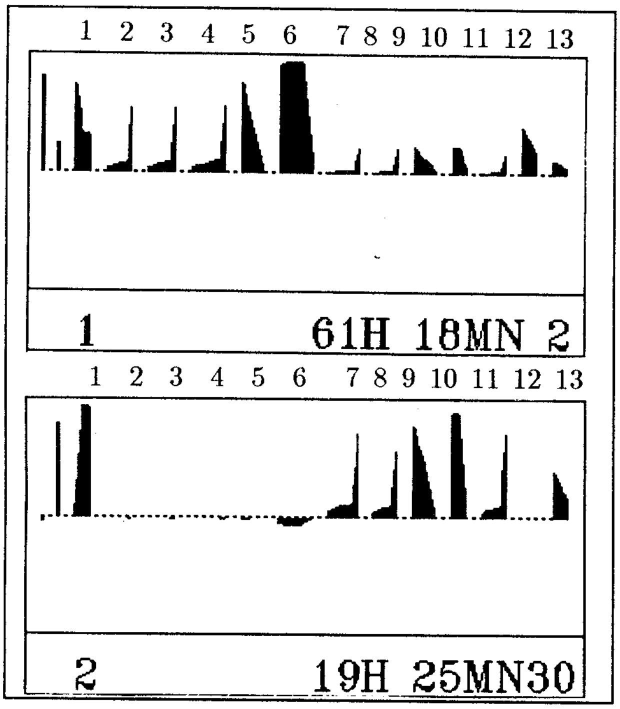

Figure 15: eigenmodes for the global modal model

5 Conclusion

These two examples, the three layers wall and the bizone building, were given as examples for the modal synthesis for two type of couplings (the edge to edge coupling and the overlapping coupling). They showed the good performances of such method and therefore open the field to new perspectives towards building thermal design.

References

Bourquin, F., "Component mode synthesis of second order elliptic operators" *C. R. Acad. Sci. Paris, t.309, Serie I, p 919-922, 1989*

El Khoury, K. and Neveu, A., "Analyse modale des systèmes thermiques en présence de transferts non réciproques", *Int. J .Heat Mass Transfer, Vol.32, n2, pp 213-226, 1989*

Gise/Cenerg, "Le projet SYMBOL : SYnthèse Modale et Boîte à Outils Logiciels" *Rapport Afme/Armines 1988/89, 1990*

Imbert, JF., " Analyse des structures par éléments finis" *Cepadues Editions, 1984*

Lefebvre, G., "Caractérisation de l'inertie thermique d'un bâtiment par analyse modale" *Rev. Gen. Therm. n 332,333, 1989*

Lefebvre, G.; Neveu, A.; El Khoury, K.; Salgon, JJ.; "Applying the modal method to thermal modelling" *In proceedings of the 1990 International Heat Transfert Conference (Jerusalem), vol.3 p 157*

Neirac, FP., "Approche théorique et expérimentale des modèles réduits de comportement thermique de bâtiments", *Thèse de doctorat, Ecole des Mines de Paris, 1989.*

Neveu, A.; El Khoury, K. ; Lefebvre, G. ; Salgon, JJS. ; "Thermique des enveloppes", *Rapport final AFME/ARMINES 1986, 1987.*

Peuportier, B. and Blanc Sommereux,I., "Simulation tool with its expert interface for the thermal design of multizone buildings". *International Journal of Solar Energy, 8,109-120, 1990.*