INTERMITTENT HEATING SYSTEM CONTROL BASED ON THE QUADRATIC OPTIMIZATION PRINCIPLE

H. ZHAO (Academy of Air Conditioning/China), JC. VISIER (CSTB/France)

ABSTRACT

This paper deals with an optimal control algorithm which enables to regulate the heating installation of a building with discontinuous occupation. The control structure is based on the quadratic optimization principle. It seems to have several characteristics that are worth mentioning, in particular its simple calculating method and its easy installation. As opposed to the optimal control algorithms based on the minimum principle, the above-mentioned algorithm can be set up in microprocessors of very low capacity. This should make it possible to incorporate it easily into building energy management systems.

One optimal control model using this algorithm has been developed and tested by simulation. The results obtained showed that this control model was able both to forecast the preheating time precisely and to control perfectly the temperatures of the buildings when occupied, whatever the external temperature or the maximum heating installation power. Simulations for a specific heating season have also been performed by using respectively the optimal control and one intermittency scheduler (the pre-heat phase starts at the same time all over the year) associated with a P.I. controller. As compared to the latter, the optimal control provides not only **more reliable comfort, but** also high energy conservation. Furthermore, the method described above makes it possible for any experienced user to elaborate rapidly optimal control models suitable for many different types of systems.

INTRODUCTION

A thermal control system for buildings is intended both to minimize the power consumption and to maintain comfort. In the case of heated buildings with discontinuous occupation, or in cases where the energy cost is not constant during the day, where one can refer to several energy sources, it is very hard to determine a control principle aimed at optimizing the operation of the given installation. Therefore, a powerful control strategy based on the optimal control principle may be envisaged.

Several studies regarding the optimal control of heating systems have already been undertaken in France, for instance within the IRCOSE (LEBRU 1987), (ROSSET 1986), (VINOT 1989) group which consisted of the "Agence Française pour la Maîtrise de l'Energie" (French Energy Control Agency), the "Ecole des Mines" (Mining Academy), "Gaz de France" (French Gas Supply Company) and the CSTB. The work was mainly focused on the use of the optimal control methods based on the minimum principle. It enabled to devise highly efficient methods for the behaviour of installations. However, the disadvantage of such methods lies in the fact that they require optimization at each time step and that they cannot be implemented on computers other than the ones with a high calculating capacity power. In this paper, we present a new control system based on the quadratic optimization principle,. This system makes it possible for us to determine a control structure by state feedback return for the multidimensional linear systems. It is aimed at reaching the "best compromise" between two opposed requirements : the difference in the output variables and energy saving. Its main advantage lies in its simple control structure. Moreover, the development of the algorithm together with the perfecting of the control may be undertaken easily.

DEFINITION OF THE CONTROL STRUCTURE.

General formulation

We consider a linear system which is represented by a discrete state model, which may be subjected to any control or observation. This model has the following formula :

$$\begin{aligned} X(i+1) &= A.X(i) + B.U(i) + F.P(i) \\ Y(i) &= C.X(i) \end{aligned} \qquad (2\text{-}1)$$

with
- $X(i)$: state vector (dimension n)
- $Y(i)$: output vector (dimension m)
- $U(i)$: input vector (dimension r)
- $P(i)$: disturbance vector (dimension r1)
- A,B,C,F : constant matrices

If e(i) represents the difference between the index value of controlled variable Z and the output Y(i), the optimal control desired, which is thus a sequence of vectors called U(i), shall minimize a criterion called J as follows :

$$J = \sum_{i=1}^{N} [\, e(i)^T.Q.e(i) + U(i)^T.R.U(i) \,] \qquad (2\text{-}2)$$

Penalization matrices Q and R are symmetrical and defined as positive.

Several theories enable to solve this problem. Below is the optimal control strategy based on the "optimality principle which provides a simple and rather "intuitive" resolution.

First, equation (2-1) shall be divided into two parts so that the disturbances may be taken into consideration more easily and so that the control performances may be improved. The first part consists of the model of the initial system without any disturbance and shall be expressed as follows :

$$\begin{aligned} X_0(i+1) &= A_0.X_0(i) + B_0.U(i) \\ Y_0(i+1) &= C_0.X_0(i) \end{aligned} \qquad (2\text{-}3)$$

The second part consists of the model which defines the effects of the disturbances on the initial system. Measurable disturbances alone are dealt with in this paper. This model shall be expressed as follows :

$$X_1(i+1) = A_1.X_1(i) + B_1.P(i)$$
$$Y_1(i+1) = C_1.X_1(i) \qquad (2\text{-}4)$$

with $X_1(i)$ which represents the supplementary state vector of n1 size.

According to the "optimality principle", optimal control vector U(i) may thus be expressed as follows (FOULARD 1987) :

$$U(i) = U_0(i) + U_1(i) \qquad (2\text{-}5)$$

with
$$U_0(i) = -L_0(i).X_0(i) + H_0(i).Z \qquad (2\text{-}6)$$
$$U_1(i) = -L_1(i).X_1(i) + H_1(i).P(i) \qquad (2\text{-}7)$$

$L_0(i)$ and $L_1(i)$ are calculated by referring to the following recurring equations :

$$\begin{cases} L_0(i) = [R + B_0^T.K_0(i+1).B_0]^{-1}.\ B_0^T.K_0(i+1).A_0 \\ H_0(i) = -[R + B_0^T.K_0(i+1).B_0]^{-1}.\ B_0^T.G_0(i+1) \\ K_0(i) = A_0^T.K_0(i+1).[A_0-B_0.L_0(i)] + C_0^T.Q.C_0 \\ G0(i) = (A_0^T-B_0.L_0(i)^T).G_0(i+1) - C_0^T.Q.I \end{cases} \qquad (2\text{-}8)$$

with I which represents the unit vector.

Based on the recurring equations (2-6) and (2-8), we can determine control vector $U_0(i)$ under the starting conditions K_0 (N)=0 and G_0(N)=0, using the iterative method by going back in time. So it is with $U_1(i)$ which may be calculated by referring to the same method.

It is worth mentioning the fact that $L_0(i)$ and $H_0(i)$ only refer to the initial system, and taking the measurable disturbances into consideration can in no way modify its elements. It just completes it by referring to new terms $U_1(i)$ which take the disturbances into account.

IMPROVING THE CONTROL STRUCTURE

Compensating for the Operation Penalization Non-Linearity

The above-mentioned control strategy is "optimal" only if ones refers to the quadratic criterion (equation (2-2)).

Term $U^T.R.U$ can be expressed either by its true meaning, i.e. energy penalization, or simply by the necessity of restricting the operation amplitudes. But in some cases, we want this term to represent the energy cost whose value is proportional to the operation amplitudes. In other words, the criterion to be minimized shall be expressed as follows :

$$J_p =, \sum_{i=1}^{N} [e(i)^T.Q.e(i) + Rc.U(i)] \qquad (3\text{-}1)$$

We reformulate the quadratic criterion by using the following expression, so that quadratic criterion J may better represent criterion J_p :

$$J_c = \sum_{i=1}^{N} \{ e(i)^T.Q.e(i) + [U(i)-U_a]^T.R.[U(i)-U_a] \} \qquad (3\text{-}2)$$

Criterion J_c can thus be made approximately equal to J_p, provided that U_a is chosen adequately and that the U variations remain limited.

By using criterion Jc represented by equation (3-2), the following equation of the optimal control may be obtained by referring to the same process :

$$U(i) = -L_0(i).X_0(i)+H_2(i)-L_1(i).X_1(i)+H_1(i) \qquad (3\text{-}3)$$

In this equation, $L_0(i)$, $L_1(i)$ and $H_1(i)$ are equivalent to the ones in equation (2-8), except $H_2(i)$ which is equal to the following :

$$H_2(i) = -[R + B_0^T.K_0(i+1).B_0]^{-1}.[B_0^T.G_0(i+1) + R.Ua] \qquad (3\text{-}4)$$

$$G_2(i) = (A_0-B_0.L_0(i))^T.G_2(i+1) - C_0^T.Q.Z + L_0^T(i).R.U_a$$

Practical Use.

The use of control U(i) as described above requires the calculation of matrices $L_0(i)$, $L_1(i)$ and vectors $H_0(i)$, $H_1(i)$ at each operating time within the given time interval. Since the calculation is carried out in reverse time, we are compelled to store all the results before being able to use the control.

This is why it is particularly interesting to consider the optimization horizon as infinite or at least very large as compared with the time scale of the physical phenomena considered. The criterion to be minimized called J shall then be expressed as follows :

$$J = \sum_{i=1}^{\infty} [e(i)^T.Q.e(i) + U(i)^T.R.U(i)] \qquad (3\text{-}5)$$

Taking account of the conditions which matrices Q and R shall comply with proves (DORATO 1971) that $K_0(i)$ recurring equation (equation 2-8) converges on a single boundary solution, providing that the system may be controlled. Therefore matrix $K_0(i)$ may be considered as a constant matrix K_0 for each time considered within a finite interval. Likewise, it is almost possible to admit that vector $G_0(i)$ is equal to a constant vector G_0, provided that index value of controlled variable Z is constant (it is modified only in the form of a step at the initial moment considered). So it is with $L_1(i)$ and $H_1(i)$. This makes it possible to simplify the optimal control equation system.

Under these conditions, optimal control U(i) may be obtained from the following simple expression :

$$U(i) = -L_0.X_0(i) + S.Z - L_1.X_1(i) + V.P(i) \qquad (3\text{-}6)$$

This expression leads us to note that the values of matrices L_0 and L_1 and of vectors S and V are independent of the system states; they may therefore be calculated "off-line" once for all. The "on-line" optimal control calculation may be obtained easily from algebra of equation (3-6).

APPLICATION TO INTERMITTENT HEATING SYSTEM.

We study the application of the control based on the quadratic optimization principle to a heating system within buildings with discontinuous occupation. We then lay emphasis on the forecasting of the time at which preheating should start.

Description Of The Heating System Model.

The heating installation is very easily modelled by means of a flow instantaneously injected into the air of the building. The indoor and solar heat gains as well as the intermittence of the ventilation are not taken into account.

The simplified model of the building shall be expressed as follows :

$$X(i+1) = A.X(i) + B.U(i) + B_1.P(i)$$
$$Y(i+1) = C.X(i) \qquad (4\text{-}1)$$

with

A, B, B_1, C : constant matrices

$X^T = (T_a, T_{st})$ $\qquad U^T = (FCH, 0)$

$P^T = (0, T_{ex})$ $\qquad Y^T = (T_a, T_{st})$

where: T_a : air temperature

T_{st} : temperature of the structure

FCH : heating flow

T_{ex} : outside temperature

In order to compensate for the influence exerted by the control on the outside temperature, we divide linear equation (4-1) into two parts as follows :

$$X_0(i+1) = A.X_0(i) + B.U(i)$$
$$Y_0(i+1) = C.X_0(i) \qquad (4\text{-}2)$$

and

$$X_1(i+1) = A.X_1(i) + B_1.P(i)$$
$$Y_1(i+1) = C.X_1(i) \qquad (4\text{-}3)$$

Whence we infer :

$$X(i) = X_0(i) + X_1(i)$$
and
$$Y(i) = Y_0(i) + Y_1(i) \qquad (4\text{-}4)$$

Control Carried Out During The Period Of Occupancy.

In order to simplify the calculation, we consider this period as an infinite period of time during which the temperature index values of controlled variables are constant. That implicitly amounts to assuming that the steady state is reached at the end of the period of occupancy, which obviously is not quite true. But such an error seems negligible to us. We suppose that U_a is equal to 0 in this period. By applying the algorithm mentioned above, we may obtain the optimal control for the U_o period of occupancy :

$$U_o(i) = -L_0.X_0(i) + S.Z - L_1.X_1(i) + V.P(i) \qquad (4\text{-}5)$$

All the L_0, L_1, S and V elements are constant.

As for the system studied, we note that L_0 is equal to L_1. According to equation (4-4), the optimal control is then expressed as follows :

$$U_o(i) = - L_0.X(i) + S.Z + V.P(i)$$

with

$$L_0 = (l_1, l_2), \quad S=(s_1,s_2), \quad V = v$$

that is :

$$U_o(i) = -l_1.T_a(i) - l_2.T_{st}(i) + s_1.T_{ac} + s_2.T_{stc} + v.T_{ex}(i) \qquad (4\text{-}6)$$

where T_{ac} : air temperature set point variable

T_{stc} : structure temperature set point

It should be noted that l_1, l_2, s_1, s_2, and v values vary with the values of matrices Q and R, providing that the latter are selected; all these parameters become steady and the "on-line" optimal control calculation may be reduced to equation (4-6).

Control Carried Out During The Period Of Vacancy.

The period of vacancy leads us to face two problems. The first one lies in the forcasting of the preheating time. The thing to be aimed at is to start the thermogenerator at full power as late as possible and to bring the building back to the desired temperature right before the end of the period of vacancy. The second problem consists in preventing an extreme cooling of the building.

As opposed to the period of occupancy, comfort in the period of vacancy is no longer necessary. We therefore assume that matrix Q is a zero matrix. In addition to this, the quadratic criterion tends to restrict action quantity amplitude U, which very often results in a very long preheating period at low power. In order to avoid this phenomenon, U_a is defined as > 0 for the period of vacancy.

Because of the change in the values of matrix Q, in the U_a value and in the index values of controlled variables, we can no longer consider the period of vacancy as an infinite period of time. Therefore, the parameters of the optimal control for the given period are sequences of vectors.

With regard to the recurring equations, the optimal control shall be expressed as follows :

$$U_{in}(i) = -l_1(i).T_a(i) - l_2(i).T_{st}(i) + h(i) - v(i).T_{ex}(i) \qquad (4\text{-}7)$$

In this equation, parameters $l_1(i)$, $l_2(i)$, h(i) and v(i) vary from one step to the other ; still they remain constant for a given step. This implies that when calculating "off-line" all the parameters corresponding to each step in the period of vacancy by means of recurring equations, the calculation of optimal control U in(j) for a given step j is obtained from equation (4-7) alone, by referring to parameters l 1(j), l 2(j), h(i) and v(j) which are known and which correspond to step j.

Another modification consists in having the thermogenerator start only when the optimal control requires a power $U_{in}(i)$ greater than or equal to the maximum power of the generator, unless the control requires energy to maintain the building at a minimum temperature. This change makes it possible for the generator to work at full power and thus to obtain the shortest possible preheating period.

By changing index values of controlled variables T_{ac} and T_{stc} in equation (4-6), the energy necessary for maintaining the building at a minimum temperature called U_t may be obtained from the same equation which shall be rewritten as follows :

$$U_t(i) = -l_1.T_a(i) - l_2.T_{st}(i) + s_1.T_{ac} + s_2.T_{stc} + v.T_{ex} \qquad (4\text{-}8)$$

with the index values of controlled variables corresponding to the period of vacancy.

In other words, we may refer to three essential equations which are = equations (4-6), (4-7) and (4-8) to apply the optimal control proposed to the intermittent heating system. Optimal control U(i) as applied to each period shall be equal to the following :

$$U(i) = \begin{cases} U_o & \text{period of occupancy} \\ U_{in} & \text{period of vacancy : preheating} \\ U_t & \text{period of vacancy : maintaining a minimum temperature.} \end{cases}$$

STUDIES ON SIMULATIONS AND COMPARISONS.

Simulation : Operation Of The Optimal Control.

Our aim is to perform a simulation study of the applications of the optimal control proposed to the intermittent heating system.

The building studied is a tertiary building of 4680 m^3, with average insulation (G = 0.5 $W/m^3.k$) and average inertia (250 kg/m^2). The maximum power of the heating is equivalent to 116 kW.

Matrix Q is given two different values as for the period of occupancy. The first value enables to penalize the difference between both the air temperature and its index value of controlled variable, as well as the difference between the temperature of the structure and its index value of controlled variable. The second value corresponds to a comfort criterion which depends on the air temperature alone. As for the period of vacancy, we assume that matrix Q is a zero matrix.

The value of R is equal to or proportional to the energy cost.

U_a is equal to 0 for the period of occupancy.

As to the period of vacancy, U_a is defined according to the maximum power of the heating installation U_{max}. It is calculated by referring to the following formula :

$$U_a = U_{max} - \sqrt{U_{max} \cdot U_m} \qquad (5\text{-}1)$$

where : U_m represents the power necessary for maintaining the building at the desired temperature, provided that the outside temperature is equal to the average outside temperature of the heating season.

All the simulations were carried out for a period from Thursday 4 p.m. to Monday 6 p.m., thus including a weekday and a weekend. In order to make the analysis easier, the outside temperature is considered as constant. The index values of controlled variables of the air and structure temperatures are 19°C for the period of occupancy and 8°C for the period of vacancy.

Figure (5-1) represents the scenario of the optimal control with weighting of the temperatures of both the air and the structure. We may note that the optimal control can forecast the preheating time accurately. The thermogenerator is restarted at full power up to the beginning of the period of occupancy. In the case where the generator does not work all night long, the air temperature at the end of the period of vacancy is higher than the index value of controlled variable 19°C. Indeed, the control is intended to adjust the temperature of the structure to 19°C by overheating the air. In the case where the generator is off during the weekend, the air temperature reaches 19°C at the end of the period of vacancy, whereas the temperature of the structure is below the one corresponding to an off-position of the generator all night long; still comfort remains reasonable. During the period of occupancy, the control is such that the air temperature remains slightly higher than 19°C and the temperature of the structure, which is below the index value of controlled variable at the beginning of the period of occupancy, gets little by little closer to the above-mentioned one. We can note as well that the control is able to maintain the minimum temperature perfectly during the period of vacancy.

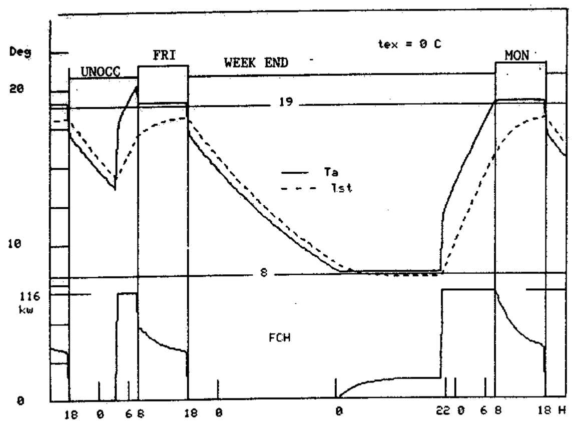

Figure (5-1) : optimal control
Air temperature Ta and structure temperature Tst
Heating flux FCH (maximum power = 116 kW)
Constant outside temperature 0°C
from Thursday 4 p.m. to Monday 7 p.m.

Influence Of The Power Of The Heating Installation.

The power of the heating installation has an influence on the rate of the building preheating. In the optimal control strategy, this power is referred to when calculating U_a (see equation (5-1)).

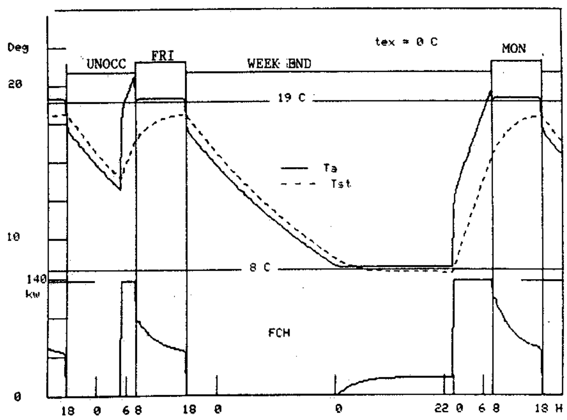

Figure (5-2) : Influence of the heating installation power
Umax = 140 kW.

Figure (5-2) corresponds to the case where the power is equal to 140 kW. The outside temperature is still 0°C. Under these conditions, the simulation result shows that the preheating time is shorter than the one mentioned in figure (5-1), because in this particular case, the maximum power is higher. Therefore, we can notice that taking the installation power into account by referring to Ua makes it possible for the optimal control to behave properly if the power of the installation varies.

It should be added that the change in the power of the heating installation can in no way modify the calculation of the optimal control for the period of occupancy, since during this period, U_a remains equal to 0.

Benefit Of Compensations For Measurable Disturbances.

The optimal control proposed enables to compensate for the measurable disturbances. In the case of a heating system intended for buildings, the measurable disturbances consist in the outside temperature and in the solar supply. In this particular case, the influence exerted by the outside temperature is just taken into account. As for the calculation of the optimal control, this influence is compensated for by the following term : [v(i).Tex] (see equation 4-7).

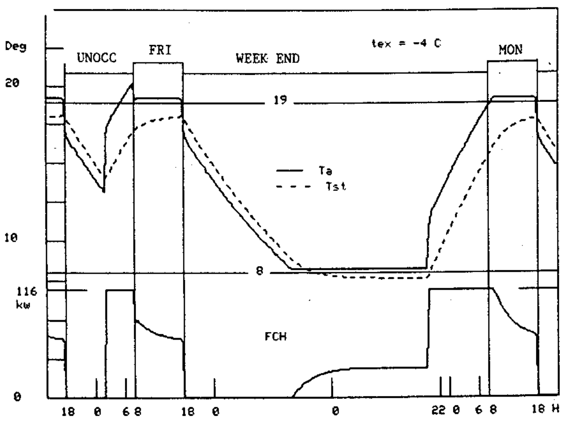

Figure (5-3) : Optimal control with compensation for the outside temperature (Tex = - 4°C).

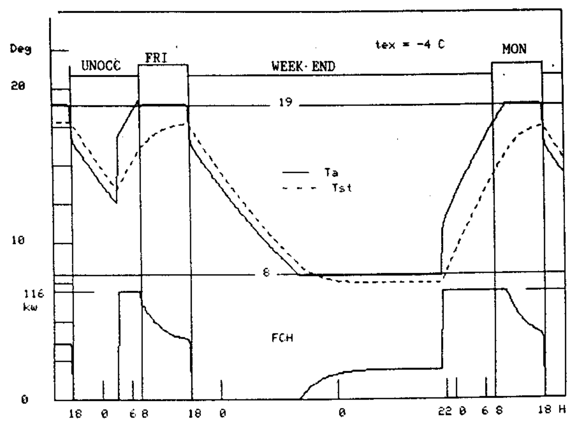

Figure (5-4) : Optimal control without compensation for the outside temperature (Tex = - 4°C).

Figures (5-3) and (5-4) represent scenarios of the optimal control for an outside temperature of - 4°C. Figure (5-3) corresponds to the control with compensation for the outside temperature. We may notice that the preheating is perfect. On the contrary, as for the optimal control without compensation for the outside temperature (figure 5-4), the thermogenerator is restarted almost at the same time as in the case where the outside temperature is equal to 0°C. Preheating is thus insufficient. We then may observe that the air temperature hardly reaches 19°C after an off-position of the generator during the whole night, whereas it just drops to 17.7°C after a stopping of the generator during the whole weekend.

Influence Of The Comfort Criterion.

The comfort criterion as applied to all the simulations mentioned above depends on the temperatures of both the air and the structure. This is the reason why the air temperature is in most cases above 19°C, because the optimal control tends to increase the temperature of the structure up to the index value of controlled variable. If we refer to a comfort criterion which depends on the air temperature alone, the situation will be different. Figure (5-5) provides the simulation result based on this criterion and including an outside temperature equal to 0°C. We may observe that instead of exceeding the index value of controlled variable at the end of the period of vacancy, the air temperature reaches then 19°C exactly, since the difference in the structure temperature is not to be regarded anymore.

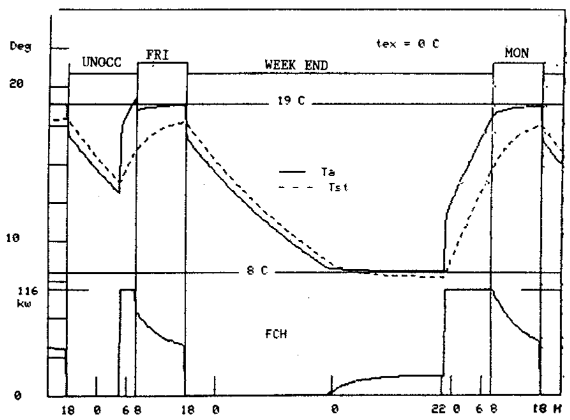

Figure (5-5) : Optimal control with comfort criterion based on air only (Tex = 0°C).

Consideration should be given to the fact that during the whole period of occupancy, the air temperature is slightly lower than the index value of controlled variable. Because the optimal control proposed is intended to reach the best compromise between comfort and energy conservation. Therefore, it tries both to save as much energy as possible and to preserve suitable comfort.

Comparison With The Usual Control For a Given Heating Season.

In order to assess the energy savings which the optimal control gives the intermittent heating system the benefit of, we compare the optimal control simulation results with the result obtained from simple intermittent heating provided by means of a clock designed to modify the set point of a PI regulator. The simulations are carried out for a whole heating season (from September 26 to April 24).

As for the optimal control, the comfort criterion depends on the air temperature alone. The outside temperature referred to is listed in a climatic file whose data are the ones obtained in the Paris area.

The simulation of a simple intermittent heating is undertaken under the same conditions (i.e. same building and same outside temperature) as the ones with optimal control. The preheating time is equal to 13 hours if the generator was off during a whole weekend and to 5 and a half hours if the generator was off all night long. These preheating times may be obtained by using the optimal control, assuming that the outside temperature is constant and equal to the minimum daily average temperature.

Figure (5-6) provides a histogram of the air temperature during the periods of occupancy, in the case where the optimal control is used. The average air temperature is 19.08°C, the highest air temperature is 19.28°C, and the lowest is 18.43°C. It may be observed that comfort is totally ensured. In addition, there is almost no overheating of the building during the whole heating season, which results in high energy savings.

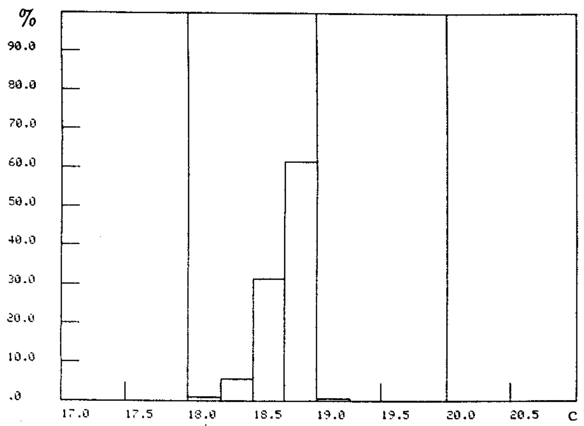

Figure (5-6) : Air temperature during the period of occupancy during a heating season optimal control.

On the contrary, the air temperature is in most cases above 19°C during the period of occupancy in cases where one uses the simple programmer equipped with a PI regulator (see figure 5-7). The average air temperature is 19.57°C, the highest is 25.8°C, and the lowest is 17.3°C.

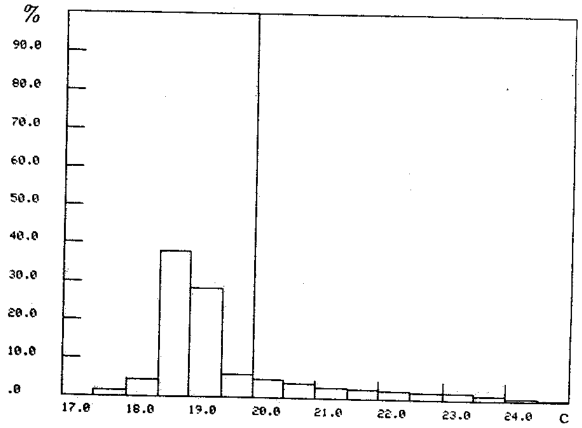

Figure (5-7) : Air temperature during a heating season simple programmer.

As for consumptions, the ones obtained by using the optimal control are by 13% lower than the ones obtained by using the simple programmer (13330 kW as opposed to 15290 kW). One may observe that a simple programmer uses far more energy during the period of vacancy (12270 kW as opposed to 5630 kW for an optimal control). This means that the optimal control is able to forecast the preheating time accurately during the whole heating season and thus prevent any unnecessary energy consumption.

CONCLUSION

We have studied an optimal control structure based on the quadratic optimization principle. The optimal control structure proposed seems to us to present several characteristics of high interest, in particular its easy calculating methods. As a matter of fact, once the process has been identified and the structural elements have been calculated "off-line", algebra alone is needed for an "on-line" calculation of the optimal control. The latter does not require much memory space; in addition to this, the calculating time needed is very short. The control can be implemented very easily. Another advantage of this optimal control method lies in its adaptability. It can adapt to the different situations by adding the supplementary terms (for instance : compensation for measurable disturbances).

The application of this method to a heating system intended for a building with discontinuous occupation has been studied by simulation. The results show that the optimal control proposed can forecast the preheating time accurately and control the temperatures of the building perfectly during the period of occupancy. It can as well maintain the minimum temperature of the building during the period of vacancy. Moreover, the control may compensate for the influence exerted by the outside temperature and adapt to the change in the heating installation power.

The simulations for a given heating season were performed by using respectively the optimal control and a simple programmer equipped with a PI regulator. In both cases, comfort in the period of occupancy is ensured. But the optimal control provides an energy conservation of approximately 13% as opposed to the simple programmer.

BIBLIOGRAPHY

DORATO P., LEVIS A.,H., 1971, "Optimal linear regulator, the discrete-time case" I.E.E.E. Trans. Autom. Control, vol. AC-16 Dec., 1971.

FOND M., 1971, "Contribution à la synthèse de la commande dynamique de systèmes multidimensionnels" Thèse de docteur-ingénieur, Université Scientifique et Médicale de Grenoble, France 1971.

FOULARD C., GENTILS J.P., SANDRAZ J.P., 1987, "Commande et régulation par calculateur numérique" Edition EYROLLES.

FRANCOIS C., HUTTER E., VISIER J.C., 1990, "Etude sur les formalismes de modèles simplifiés de bâtiment" Rapport GEC N° 89.4971 Mars 1990.

LEBRU A., VISIER J.C., 1987, "Commande optimale du système de chauffage électrique d'un immeuble : mise en oeuvre et expérimentation" Third International Congress on Building Energy Management, PPR, Lausanne 1987.

ROSSET M.M., 1986, "Gestion thermique optimale d'un bâtiment" Thèse de docteur, Université de Paris-sud Centre d'orsay, 1986.

SANDRAZ J.P., 1973, "Identification et commande multidimensionnelles d'une unité pilote de fabrication de papier" Thèse de docteur-ingénieur, Université Scientifique et Médicale de Grenoble, France 1973.

VINOT B., 1989, "Commande optimale et/ou adapdative des systèmes énergétiques" Rapport AFME, Septembre 1989.

VISIER J.C., D'ACREMONT E., NIBEL S., CACCAVELLI D., HUTTER E., VINOT B. (CSTB, France) and PICARD P. (Gaz de France), 1990, "Heating control strategies in non-permanently occupied buildings". Third International Conference on System Simulation in Buildings" Liège, Belgium December 1990.

VISIER J.C., 1988, " Pratique de l'intermittence du chauffage dans les locaux à occupation discontinue" Cahiers du CSTB, Octobre 1988.