THREE-DIMENSIONAL MODELLING OF SLAB-ON-GRADE HEAT TRANSFER

by

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ABSTRACT

Experimental studies during the 1940S concluded that heat loss from slab-on-grade floors is proportional to floor perimeter length. More recent numerical investigations, however, indicate that area and shape are also important parameters. Furthermore, results of three-dimensional modelling differ significantly from those of supposedly equivalent two dimensional analysis. Earth-coupled heat transfer processes are increasingly important contributors to building energy consumption, but continue to be poorly understood by most designers.

The twin objectives of the present study were to increase fundamental understanding of threedimensional effects on slab-on-grade heat transfer and to provide validation data for simplified models. Over ninety hourly, annual simulations were performed with a detailed three-dimensional finite difference model. Parametric studies considered effects of geometry, climate, soil properties, and boundary conditions. The results of this study show that floor heat flux depends on the ratio of floor area to floor perimeter length (A/P). A potential manual method employing this scaling relationship is described. This model provides a means for characterizing the degree of area dependence of floor heat transfer as a function of building and environmental parameters. A companion paper reports on the parallel development of an improved slab-on-grade model compatible with transfer function based energy analysis programs such as BLAST.

NOMENCLATURE

A	Area [m^2]
F_2	Perimeter heat loss coefficient [W/m-K]
G	Rate of conduction to ground [W/m^2]
K_1, K_2	Simplified model conductances [W/m^2-K]
L	Length [m]
P	Perimeter length [m]
Q	Heat transfer rate [W/m^2]
R_t	Net radiation absorbed at ground [W/m^2]
T	Temperature [C]
c	Constant in heat flux model
d	Exponent in heat flux model
h_i	Convection/radiation coefficient [W/m^2-K]
k	Thermal conductivity [W/m-K]
q	Heat flux [W/m^2]
q_{cs}	Sensible convective flux
q_{et}	Evapotranspiration flux
ϕ	Phase lag of floor heat flux [days]
ρc	Heat capacity [J/m^3-K]

INTRODUCTION

The impact of earth-coupled heat transfer processes on the energy consumption and thermal comfort of buildings has been studied by building scientists for more than forty years [1, 2]*. At the time of these early investigations, the uninsulated basement of a typical home might have accounted for as little as ten percent of its total energy consumption. Because of improved standards for insulation and infiltration, however, the same basement could be responsible for half the energy consumed by a comparable contemporary structure [3]. Consequently, the need for accurate prediction of earth-coupled building heat transfer is greater today than ever before.

However, progress toward the development of accurate, flexible methods for earth-coupled heat transfer analysis has been slow. MacDonald, Claridge, and Oatman recently reported disagreements as large as 1000% between seven simplified methods for basement heat loss [4]. Prominent defects of most models are limited consideration of soil property effects, *ad hoc* assumptions about the importance of three-dimensional effects, and questionable representations of the ground surface boundary condition. In general, these deficiencies in design methods are attributable to gaps in basic research.

In the past decade, numerical models have assumed an important role in earth-coupled heat transfer research [5-10]. In many respects, however, the body of knowledge generated through modelling is incomplete. Most prior models have been two-

* Numbers in brackets refer to entries in REFERENCES.

dimensional, leaving unanswered questions concerning the nature and magnitude of three-dimensional effects. (The few three-dimensional studies on record indicate 20-50% discrepancies between 2-D and 3-D results [7, 9].) Ground-surface boundary conditions frequently have been modelled in a simplified manner. Evaporative and radiative effects generally are neglected, and smoothed approximations to weather variables usually substitute for actual data. Additionally, the relatively large time steps (as long as a week) in most transient simulations do not resolve diurnal effects that may be important influences on thermal loads and comfort.

In order to address some of these issues, the present study of slab-on-grade floors employed a finite difference model (FDM) containing significantly fewer *a priori* simplifying assumptions than most of its predecessors. The most important features of this model were its three-dimensional geometry, hourly time step, detailed ground surface boundary condition, and use of TMY weather data. It was expected that this high level of detail would make it possible to validate some of the assumptions commonly employed in other models.

A further objective was to reexamine the widely held notion that slab-on-grade heat transfer is proportional to perimeter length. This assumption is central to the "F_2" heating load method currently recommended by ASHRAE [11]:

$$Q = F_2 \cdot P \cdot \left(T_{inside} - T_{outside}\right) \tag{1}$$

Eqn. 1 states that the heat loss from a slab floor is equal to the product of its perimeter length, the indoor-outdoor air temperature difference, and the climate and construction dependent factor "F_2". This model was proposed in the 1940s on the basis of experimental measurements [1, 2] and later updated with new coefficients derived from the finite element model of Wang [5]. The basis of the F_2 method was tested by extensive parametric studies of area and aspect ratio effects.

PROBLEM AND SOLUTION

This section outlines the formulation and numerical solution of the three-dimensional slab-on-grade problem developed in this study. The interested reader may consult reference [12] for a more detailed discussion.

For purposes of a generic study, energy transport in the soil may be approximated as a boundary value problem on the heat conduction equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \cdot \nabla T) \tag{2}$$

At large distances from the floor perimeter, the temperature distribution is that of the undisturbed ground, i. e.:

$$T = T(z, t) \tag{3}$$

Depending on circumstances, either a zero-flux or fixed temperature condition may be more appropriate for the deep ground, i. e.:

$$\frac{\partial T}{\partial z} = 0 \quad \text{for} \quad z \to \infty \tag{4a}$$

or

$$T = \text{constant} \quad \text{at some } z > 0 \tag{4b}$$

The typical boundary condition in this study was soil temperature at a depth of 15m fixed at the annual average dry bulb temperature.

On the inner floor surface, a linear convective-radiative condition may be assumed.

$$Q = h_i \cdot A \cdot \left(T_{room} - T_{floor}\right) \tag{5}$$

The h_i values recommended by ASHRAE were used. The indoor air temperature, T_{room}, was held constant at 22 C throughout the year.

A gradient condition at the ground surface may be derived from the energy balance of Eqn. 6 [13], which states that the net rate of conduction into the ground ($G = -k(\partial T/\partial z)$) is equal to the net absorbed radiation flux (R_t) less sensible convective (q_{cs}) and evapotranspiration (q_{et}) losses to the atmosphere.

$$G = R_t - q_{cs} - q_{et} \tag{6}$$

This boundary value problem was discretized and solved numerically by the Patankar-Spalding finite difference method, a well-known and widely accepted approach [14]. The solution employed an hourly time step and was driven by TMY weather data.

All floors were 10cm thick concrete slabs. Plan areas ranged from 144m^2 to 3600m^2. Most cases were of rectangular shape. Aspect ratios for rectangular cases varied from unity (square) to 9 (20m x 180m). Several L-shaped plans were modelled in order to determine whether there is a significant shape effect on slab floor heat transfer.

Besides geometric factors, parametric studies considered the effects of climate (four locations), soil thermal properties (five sets), deep ground boundary condition, ground surface conditions, building ground shadow, and beneath-slab insulation (two thicknesses in two configurations). In all, 93 hourly, annual simulations were performed.

RESULTS

Geometric Scaling

It was found that slab-on-grade heat transfer deviates substantially from the widely accepted law of linear dependence on perimeter. For example, consider the comparison of daily averaged heat loss per unit perimeter length for uninsulated floors of different areas in Medford, OR shown in Fig. 1. Although the two curves are of similar shape, they are offset by roughly 7-8 W/m. This difference in mean heat loss is due to the difference in area of the floors and causes the floor with greater area to lose more heat per unit perimeter length than the smaller floor. In consequence, F_2 factors based on the heat loss of the 12 x 12m floor would be low by 25-50% (depending on the season) when applied to the estimation of heat loss from the 45 x 45m floor.

Further evidence of problems with the F_2 method is given in Figure 2, which shows annual average heat loss as a function of perimeter for twenty runs in a 15 m deep domain using Medford, OR weather. Clearly, the data do not fall on a single Q vs. P curve. Heat loss increases with increasing area beyond the amount predicted by a "proportional to perimeter" model. These results include both rectangular and L-shaped floor plans. L-shaped cases fall into place among rectangles of the same area (e.g., the middle point of the five 2025 m^2 cases is L-shaped). Evidently, the relationship between area and perimeter, not the particular shape is more important.

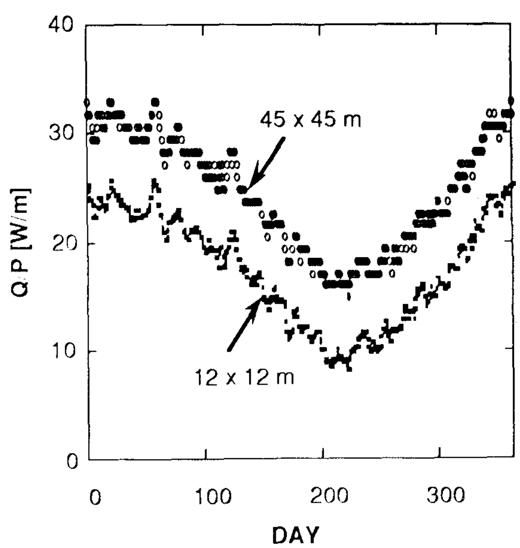

Figure 1. Effect of area on daily averaged heat loss per unit perimeter length.

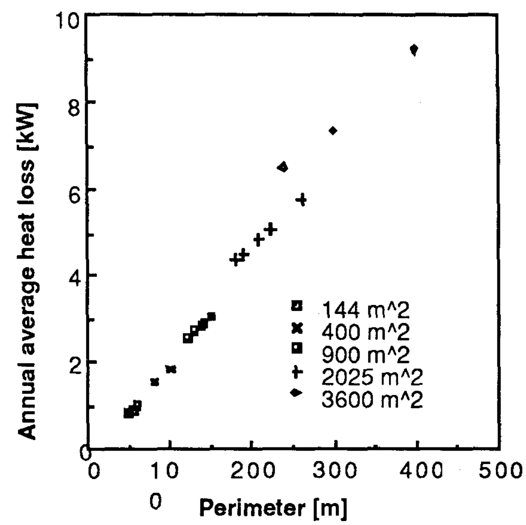

Figure 2. Effect of area on annual heat loss per unit of perimeter length.

The preceding observation suggests that A/P may be a useful length scale for slab-on-grade heat transfer. For rectangles with short side "L" and aspect ratio "μ" (defined ≥ 1), A/P is equal to L/[2(1+1/μ)]. Values of A/P for the square and infinite strip limits are, respectively, L/4 and L/2. When the data of Fig. 2 are replotted as annual-averaged heat loss per unit area vs. A/P, all of the data fall on a single curve, as shown in Figure 3. This curve may be approximated by the logarithmic function:

$$q = c \cdot \left(\frac{A}{P}\right)^{d} \qquad (7)$$

where c and d are constants. The A/P dependence of q in Eqn. 7 implies that the total heat transfer rate, Q, is linearly proportional to perimeter when d = -1 and linearly proportional to area when d = 0.

Two points should be kept in mind concerning the model of Eqn. 7. First, the constants "c" and "d" depend on several parameters, including climate, soil properties, domain geometry, and details of foundation design. The values of these coefficients can be expected to change with conditions. Secondly, the fluctuating component of the heat flux will not necessarily behave in the same way as the mean flux.

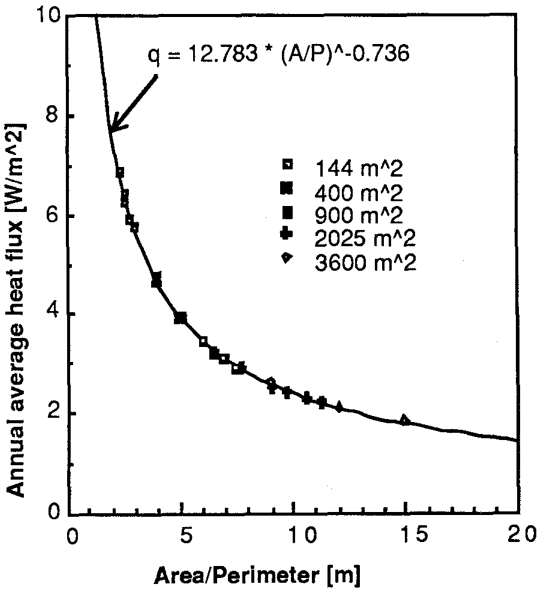

Figure 3. Relation of annual heat flux to floor area-perimeter ratio.

Simplified Model of Daily Averaged Heat Flux

This model of annual heat transfer may be extended to the total heat transfer rate by separating it into mean and fluctuating parts:

$$q_{total}(t) = q_{mean} + q_{periodic}(t) \tag{8}$$

If one assumes that the mean heat loss is proportional to the difference between the indoor air and outdoor ground surface temperatures while the periodic loss is a function of the difference between the daily averaged and annual mean ground surface temperatures, then:

$$q_{total}(t) = K_1 \cdot \left(T_{room} - T_{g,\,mean}\right) + K_2 \cdot \left(T_{g,\,mean} - T_{g,\,\phi}\right) \tag{9}$$

Where K_1 and K_2 are constant mean and periodic conductances (SI units W/m^2-K). $T_{g,\,\phi}$ is the ground surface temperature lagged by an empirical phase constant, "ϕ". Ground surface temperature more accurately represents conditions in the soil near the slab under varying surface conditions than does air temperature. The phase lag ϕ accounts for the possibility that soil mass beneath a slab-on-grade may delay the effect of above-ground conditions.

The ground temperature, T_g, may be approximated by a sinusoidal least squares model of soil temperature data :

$$T_g = T_{g,\,mean} + \Delta T_g \cdot \sin\left(2\pi \frac{(Day + \zeta)}{365}\right) \tag{10}$$

where ΔT_g is the amplitude of the annual ground temperature cycle, "Day" is the day of the year (1-365), and ζ is the phase shift (in days) of the ground temperature with respect to the calendar. $T_{g,\,\phi}$ differs from T_g only by virtue of the additional phase shift:

$$T_{g,\,\phi} = T_{g,\,mean} + \Delta T_g \cdot \sin\left(2\pi \frac{(Day + \zeta + \phi)}{365}\right) \tag{11}$$

The geometric dependence of K_1 and K_2 for arbitrary floors is presumed to be the same as that in Eqn. 7:

$$K_1 = c_1 \cdot \left(\frac{A}{P}\right)^{d_1} \tag{12a}$$

$$K_2 = c_2 \cdot \left(\frac{A}{P}\right)^{d_2} \tag{12b}$$

With substitution from Eqns. 11 and 12, the complete daily averaged heat flux model of Eqn. 9 becomes:

$$q_{total}(t) = c_1 \cdot \left(\frac{A}{P}\right)^{d_1} \cdot \left(T_{room} - T_{g,\,mean}\right) - c_2 \cdot \left(\frac{A}{P}\right)^{d_2} \cdot \Delta T_g \cdot \sin\left(2\pi \frac{(Day + \zeta + \phi)}{365}\right) \tag{13}$$

Values of the constants c_1, c_2, d_1, and d_2 are determined by a two stage process. First, K_1 and K_2 values are calculated for several floor A/P values by least squares approximation of daily averaged heat flux results. Then, c_1, c_2, d_1, and d_2 are obtained by a second round of approximations using Eqns. 12. For the Medford, OR cases considered previously, the values of c_1, c_2, d_1, and d_2 were, respectively, 0.978, -0.747, 0.713, and -0.999. This value of d_2 indicates that the time-varying component of heat flux is essentially proportional to perimeter--unlike the mean component, which has significant area dependence. The phase lag was found to vary little from one floor to another, so an average value of ϕ may be used in Eqn. 13. (Values observed in this study were on the order of 18 days.)

Daily averaged heat flux results and the approximate model "q_{total}" for a 12 x 12 m uninsulated slab are compared in Figure 4. The agreement between the two is quite good considering that a smoothed approximation of ground temperature was used in place of the actual ground temperature.

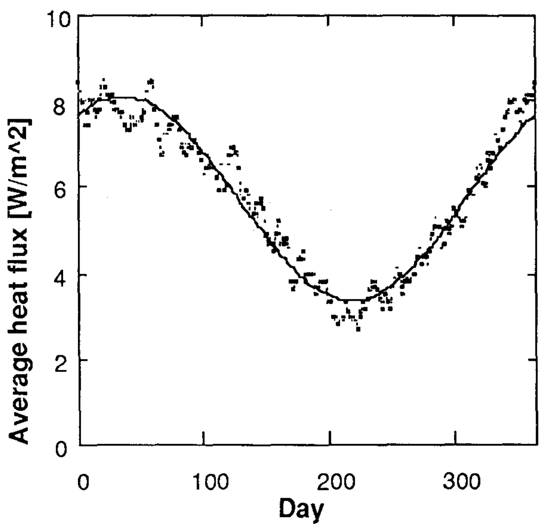

Figure 4. Comparison of daily flux model and data for a 12 x 12 m slab in Medford, OR.

Climate Effects

Assessment of climatic influences was based on simulations of four rectangular uninsulated slabs in each of four climates: Medford, OR; Minneapolis, MN; Philadelphia, PA; and Phoenix, AZ. Mean air temperatures for these sites ranged from 7 C to 21.8 C, while centigrade heating degree days varied from 773 to 4636. All had potential evapotranspiration ground surface conditions and mean outside dry bulb fixed temperature lower boundary conditions. Following the approach of the previous section, least squares models of daily averaged heat flux were computed and their coefficients were compared.

Table 1 gives coefficients of daily averaged heat flux models for the four locations. These indicate strong similarity between results for Medford, Minneapolis, and Philadelphia. In all four cases, the time varying component of heat loss is linearly proportional to perimeter length and independent of area ($d_2 \approx -1.0$). The primary difference between Phoenix and the other sites is the degree of area dependence of the steady state heat transfer component (much weaker for Phoenix). Exponent d_1 may be larger for Phoenix because the deep ground and indoor temperatures are nearly identical (22 C vs. 21.8 C). When there is no mean temperature difference between the floor and the deep ground, mean loss must be toward the ground surface from the floor perimeter. In this limit, the mean loss should depend on perimeter in a manner similar to the periodic loss. Because model coefficients are similar across a range of climates, it is reasonable to conclude that differences in climate are adequately represented by the mean and amplitude of ground surface temperature used in this.model.

Table 1. Daily averaged heat loss model coefficients for climate variation tests.

Location	c_1	d_1	c_2	d_2
Medford, OR	0.978	-0.747	0.713	-0.999
Minneapolis, MN	0.997	-0.735	0.759	-0.999
Philadelphia, PA	1.007	-0.750	0.765	-0.995
Phoenix, AZ	1.041	-0.901	0.769	-0.997

Other Parametric Studies

Effects of other parameters, including soil properties, insulation, and surface boundary conditions were examined through the use of the proposed simple model. It was found that the area dependence of floor heat flux increases as soil conductivity increases, but is insensitive to soil thermal diffusivity. Likewise, adding insulation also increases area dependence of the mean heat flux. The latter observation suggests that the F_2 method is less accurate for insulated foundations than for uninsulated. Details of these and other parametric investigations may be found in [12].

CONCLUSIONS

This paper has described an analysis of slab-on-grade heat loss characteristics based on an extensive program of detailed simulations. Several conclusions follow from this analysis:

- Slab-on-grade heat loss depends on area as well as on perimeter length. Therefore, the commonly used F_2 load calculation method is unreliable.
- A simple model which relates heat flux to the length scale A/P is a good approximation of the numerical results of this study.
- Mean and fluctuating heat flux components exhibit different degrees of area dependence, therefore they should be distinguished in models.
- The proposed A/P scaling may be useful in future modelling efforts because it permits the mapping of three-dimensional cases into equivalent two-dimensional cases. (That is, for any three-dimensional plan shape, there is a two dimensional strip with the same value of A/P.)

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