MODELLING OF THERMAL SYSTEMS FROM TECHNICAL SKETCHES TO EQUATIONS.

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ABSTRACT
Modellers ands users of simulation softwares need to agree on a standard way to state the physical bases of their models
The proposals presented in this paper are not new; they refer to the very classical way of describing thermodynamical systems. The basic piece of this description is the reference volume which may be "crossed" by mass and energy flows and which may also have some (mass and/or energy) "capacity".
R-C networks are nothing more than "degenerate" or "simplified" sets of reference volumes.
All the buildings and HVAC subsystems may be subdivided into sets of interconnected volumes. Only some elementary examples are given hereafter ...

INTRODUCTION

"How far may I trust your model? What could be the real accuracy of your results? Why don't we find a satisfactory agreement between our two models ?", etc... Many of these questions could be answered more easily or even be avoided if the modelling hypotheses had been more explicit !

Something more is needed among the technical drawings, the equivalent electrical schemes, the information flow diagrams and/or the interaction matrix.

Whatever could be the numerical resolution and simulation software, we need a standard way of declaring all the physical hypotheses dissimulated "somewhere behind" our algorithms.

Some very modest proposals are submitted hereafter; they intend to make easier the dialog between modellers and users of simulation softwares...

MASS AND ENERGY BALANCES

Our technical systems may often be subdivided into volumes of reference, on which mass and energy balances have to be established.

One mass flow rate ($\dot{M}_{ji}$) and one "global" energy flow rate ($\dot{E}_{ji}$) can be identified on a "branch" (ji) going from the reference volume i to the reference volume j of the system considered (Figure 1).

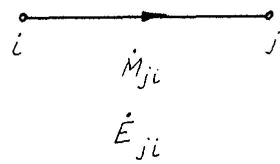

Figure 1: Mass and energy flow rates through a "branch"

The "global" energy flow rate $\dot{E}_{ji}$ can be subdivided into three components:

$$\dot{E}_{ji} = \dot{W}_{ji} + \dot{Q}_{ji} + \dot{H}_{ji} \qquad (1)$$

with $\dot{W}_{ji}$ = work flow rate (or "power")
$\dot{Q}_{ji}$ = heat flow rate
$\dot{H}_{ji} = \dot{M}_{ji}\, h_i$ = enthalpy flow rate
h_i = "massic" enthalpy of the fluid which is flowing through the branch considered.

NB: In many cases, h_i can be considered as a thermodynamic state variable. In some other cases, kinetic and potential energies might have to be taken into account and h_i become a "total" enthalpy...

According to these definitions, the "branch" linking i to j could have various physical meanings; some of them are schematized in Figure 2.

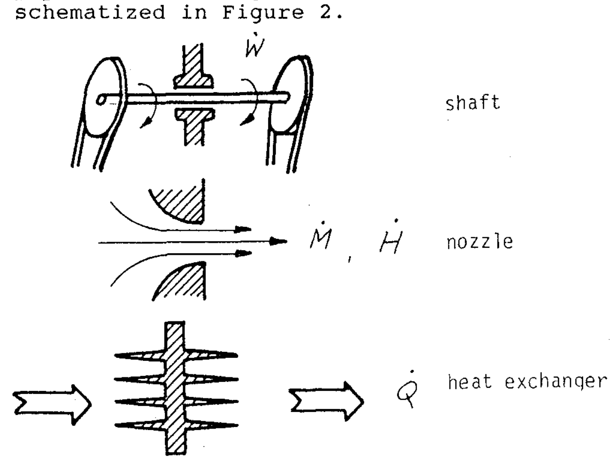

Figure 2: Possible meanings of a branch

Mass and energy balances play an essential role in the analysis of most of our technical systems :

$$\Sigma_i \dot{M}_{ji} = \delta M_j/\delta\tau \qquad (2)$$
$$\Sigma_i \dot{E}_{ji} = \delta E_j/\delta\tau \qquad (3)$$

with M_i and E_j = mass and energy contents of the volume j considered.

In many practical problems, E_j may be reduced to a thermodynamic state variable: the "internal" energy ($E_j \approx U_j$).

In other cases, variations of kinetic and/or potential energies may have to be taken into account...

At least as references (limit cases), but also as first approximations, steady state balances are always to be checked on a certain time period.

$\Sigma_i \dot{M}_{ji} \approx 0$ (2')
$\Sigma_i \dot{E}_{ji} \approx 0$ (3')

The two ways of defining the reference volume are presented in Figure 3.

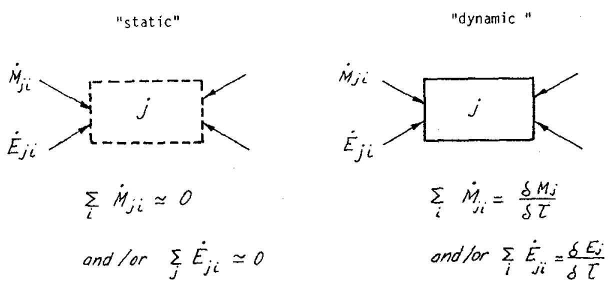

Figure 3: "Static" and "dynamic" reference volume

According to the applications considered, it may very well occur that the reference volume j has to be considered as "static" in mass and as "dynamic" in energy.

Yet at this very preliminary stage, the various meanings possible for branches (ij) and reference volumes (i,j) will justify separate sketches for description of mass and energy exchanges respectively. Without other specific indication, the mass and energy transfers ($\dot{M}_{ji}$ and $\dot{E}_{ji}$) are defined in algebraic values. They are considered as positive from i to j. But they may also be conventionnally subdivided into supply (su) and exhaust (ex) flow rates (Fig. 4).

$\Sigma_i \dot{M}_{ji} = (\dot{M}_{su})_j - (\dot{M}_{ex})_j$ (4)
$\Sigma_i \dot{E}_{ji} = (\dot{E}_{su})_j - (\dot{E}_{ex})_j$ (5)

Such conventions allow us to introduce into our sketches our a priori knowledge about the flow patterns. It contents nevertheless some danger of confusions when dealing with enthalpy flows: to a positive mass flow rate ($\dot{M}_{su}$ or $\dot{M}_{ex}$) could correspond a negative enthalpy flow rate ($\dot{H}_{su}$ or $\dot{H}_{ex}$), due to a negative value of the massic enthalpy (h) and reciprocally... The use of "supply" and "exhaust" definitions should be therefore reserved as much as possible to mass flow rates and/or to very simple situations.

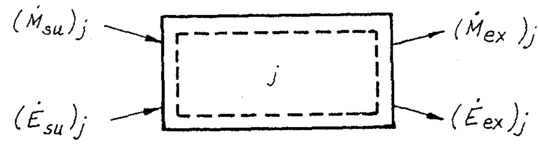

Figure 4: Conventional definition of supply and exhaust flow rates

STATE VARIABLES AND FLOW RATE POTENTIALS

In most of the practical problems we have to solve, our "hope", when defining a reference volume (j), is that its energy content (E_j) will depend only on a small number of state variables. In some very favourable cases, only one state variable (for example, the temperature T_j) would allow us to define the whole energy contents of the volume (j) considered :

$\delta E_j/\delta\tau \approx C_j \delta T_j/\delta\tau$ (6)

with C_j = "thermal" capacity of volume j

The analogy between "thermal" and "electrical" capacity concepts is used from a very long time (Figure 5).

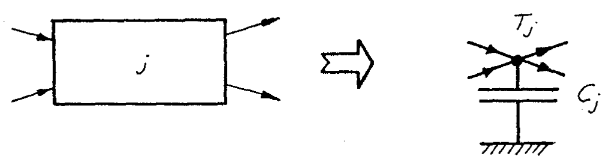

Figure 5: Reduction of the reference volume to a "thermal capacity"

There are many restrictions to the use of a reference temperature (T_j) as the only one state variable: theoretically , it would suppose that only perfect gas is contained in the reference volume (prefect liquids and solids could be accepted only at a certain degree of approximation), it would suppose also a perfect thermal equilibrium inside this volume, stability of internal movements, etc...

Another very important "hope" in our system analysis is that state variables could be used also as flow rate "potentials".

In our domain of interest, the most used potentials are pressures (p) and temperatures (T). Fluid pressure could also be used as "state variable" in some mass and/or energy balances. The use of these potentials allows to define mass flow and heat flow resistances (Figure 6).

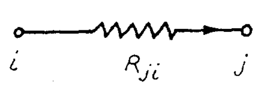

Figure 6: Resistance of a branch

$R_{ji} = (p_i - p_j)/\dot{M}_{ji}$ (7)
(Mass flow resistance)

$R_{ji} = (T_i - T_j)/\dot{Q}_{ji}$ (8)
(Heat flow resistance)

Again here the analogy with electricity is obvious. But it may occur that the resistances R_{ji} depend strongly on the potentials to which they are submitted...

"Lines" with series of resitances and capacities are often identified in our technical systems. These "lines" may also be characterized by their impedances (I) at typical frequencies ν (Figure 7).

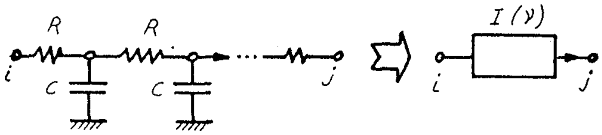

Figure 7: Impedance of a "R-C" line

$\bar{I}_{ji} = (\bar{p}_i - \bar{p}_j)/\bar{M}_{ji}$ (9)
(mass flow impedances)

$\bar{I}_{ji} = (\bar{T}_i - \bar{T}_j)/\bar{Q}_{ji}$ (10)
(heat flow impedances)

With flows and potentials represented as vectors rotating at the frequency (ν) considered.

NB: This definition has to be handled with great care: the flow rate, and therefore also the corresponding impedance, may very strongly change from one side (i) to the other one (j) along the line considered. In fact, this impedance is nothing else than the "dynamic" reference volume previously defined (Figure 3). Its characterization through harmonic analysis and equations like (9) and (10) is only one among various approaches currently used. Transfer functions or response factors are other typical ways of dealing with dynamic volumes...

CONVECTION

In the current case where there is no mass storage in the volume (j) considered, the global enthalpy flow may often be treated as an equivalent convective heat transfer (Figure 8).

$(\dot{H}_{su})_j - (\dot{H}_{ex})_j = \dot{Q}_{cj}$ (11)

This term may also be expressed as function of a potential difference:

$\dot{Q}_{cj} = (\dot{M}_{su})_j[(h_{su})_j - (h_{ex})_j]$ (12)
or
$\dot{Q}_{cj} = (\dot{C}_{su})_j[(T_{su})_j - (T_{ex})_j]$ (12')

with $(\dot{C}_{su})_j$ = "capacity flowrate" supplying the volume j (this last definition is only correct with a perfect gas; but it may often be used in good approximation also with liquids).

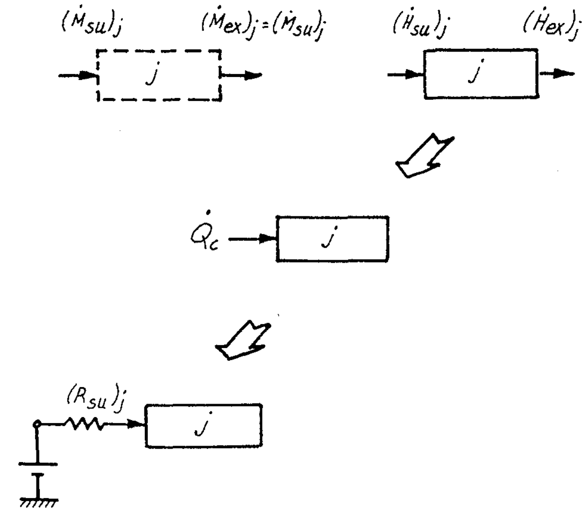

Figure 8: Definition of an equivalent convective heat transfer

The equivalent convective resistance is:

$(R_{su})_j = 1/(\dot{C}_{su})_j$ (13)

If, for example, all the mass flow rate supplying (j) was coming from a single volume (i), we would have :

$(R_{su})_j = R_{ji} = 1/\dot{C}_{ji}$ and, of course, also : $R_{ij} = 1/\dot{C}_{ij}$

The enthalpy flow is only fully equivalent to a heat transfer between i and j if :

$\dot{C}_{ij} = \dot{C}_{ji}$

i.e. if the mass flow is fully symetrical (Fig. 9), we obtain :

$\dot{Q}_{cji} = -\dot{Q}_{cij} = \dot{C}_{ji}(T_i - T_j)$ (14)

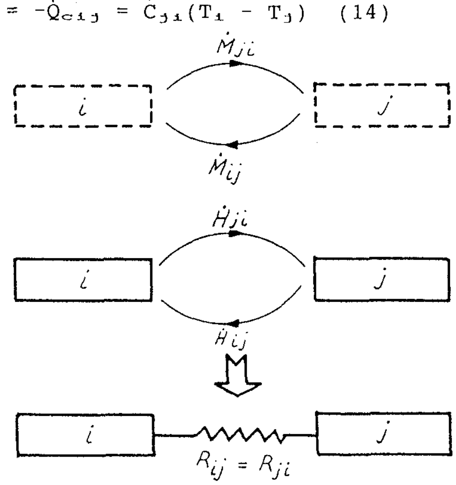

Figure 9: Symetrical convective heat transfer

NB: This representation doesn't require a perfect isothermy inside each reference volume. It is valid when T_i and T_j are defined as exhaust temperatures.

SOURCES

At least in first approach, flow and potential "sources" have to be applied in many places inside our conceptual scheme. Some of these sources are constants; some others are "controlled" by one or more other variables x,y,z,... (Figure 10).

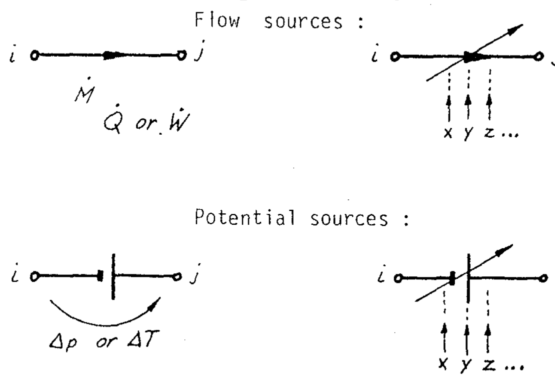

Figure 10 : Definition of sources

NB: Resistances and impedances are nothing else than variable sources when the "control" variables are potentials of adjacent volumes.

Sources statement constitutes one of the most important part of the modelling procedure. Of course, this statement may have to be reviewed at each part of an iterative simulation.

STATIC HEAT EXCHANGER (Figure 11)

A heat exchanger is generally supplied by two fluids. Each fluid channel may be treated as a reference volume. Equation (8) is directly applicable to this system :

$$\dot{Q}_{ji} = -\dot{Q}_{ij} = (1/R_{ji})(T_i - T_j) \quad (8')$$

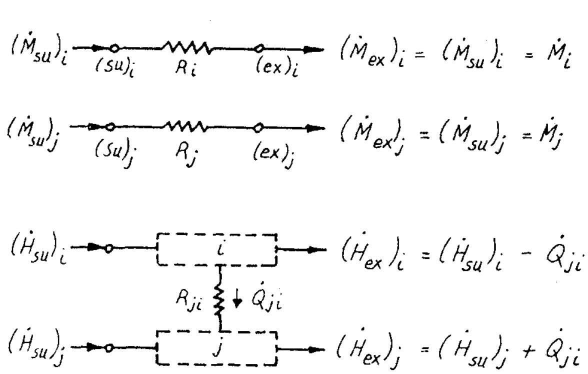

Figure 11 : Static heat exchanger

There are two classical ways of defining the heat transfer resistance and the corresponding temperature difference (T_i-T_j) :

$$\dot{Q}_{ji} = \varepsilon \dot{C}_{min}[(T_{su})_i - (T_{su})_j] \quad (15)$$
and
$$\dot{Q}_{ji} = AU\,\Delta T_{ln} \quad (16)$$

with ε = f(NTU,Ω) = heat exchanger effectivness.

NTU = $AU/\dot{C}_{min}$ = number of transfer units
AU = heat transfer coefficient
$\Omega = \dot{C}_{min}/\dot{C}_{max}$
ΔT_{ln} = logarithmic difference between temperatures of volumes i and j.

DYNAMIC HEAT EXCHANGER

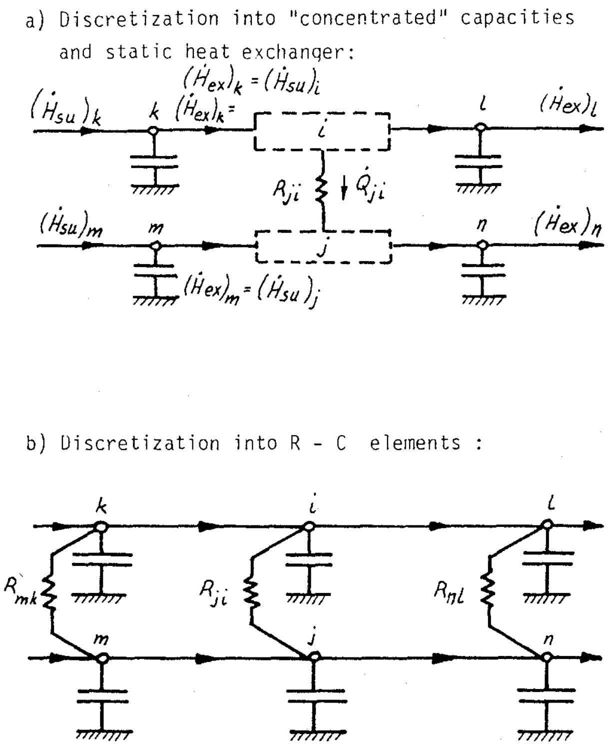

Figure 12 : Two possible ways of modelling a dynamic heat exchanger

Equations (15) and (16) could not be directly transposed to a dynamic heat exchanger; this one must be discretized into simpler components (Figure 12).

The discretization of Figure 12/a keeps unchanged the static model, but it may produce "strange" results when one mass flow has to be nullified : the capacities initially supplied by this flow will be then "disconnected" from the rest of the system !

The discretization of Figure 12/b doesn't present the same inconvenient, but it requires a more delicate definition of each thermal resistance in order to reproduce correctly the steady-state behaviour of the heat exchanger.

For example, starting from equation (15),

$\dot{Q}_{ji} = \varepsilon \dot{C}_{min}[(T_{su})_i - (T_{su})_j]$
with
$(T_{su})_i = (T_{ex})_i + \dot{Q}_{ji}/\dot{C}_i$
and
$(T_{su})_j = (T_{ex})_j - \dot{Q}_{ji}/\dot{C}_j$
We get :
$\dot{Q}_{ji} = [(T_{ex})_i - (T_{ex})_j] \ / \ R_{ji}$ (17)
with
$R_{ji} = (1 - \varepsilon - \varepsilon\Omega) \ / \ \varepsilon \dot{C}_{min}$ (18)

Of course, a thinner discretization could be considered. It could, for examples, make appear several capacities in each "slice" of the heat exchanger (Figure 13).

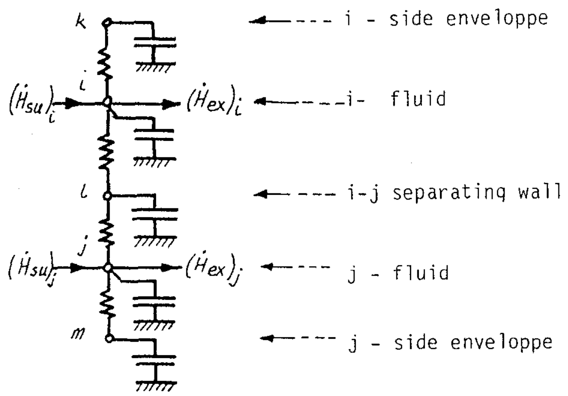

Figure 13 : "Slice" of heat exchanger with 5 capacities

N.B. : a large number of "slices" such as in Figures 12/b or 13 should be used in order to give back the "time lack" when the massflow rates are small in comparison with the mass capacities ...

DISTRIBUTION CIRCUITS

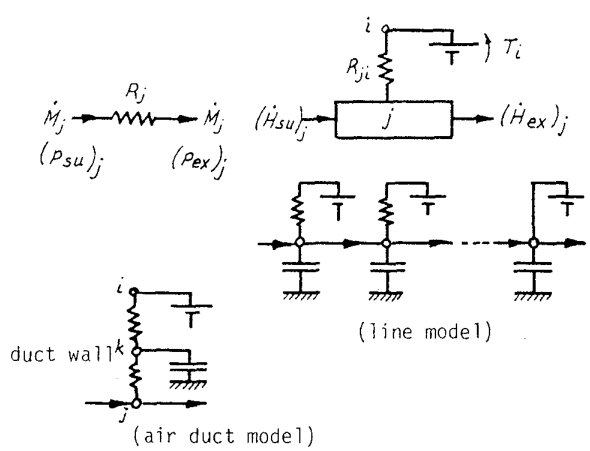

Figure 14 : Distribution models

They may behave as (dynamic) heat exchangers, the "second" fluid (i) being here the surrounding medium.

Example of general and simplified models are presented in Figure 14.

PHASE CHANGES INSIDE HEAT EXCHANGERS

A single thermal resistance would not be enough to characterize a branch in which phase change occurs.

Latent heat transfer is sometimes dealed with separately, through a heat flow source (this is the "SHR" method applied in cooling coil modelling) or through a shift from temperature to enthalpy potentials ("three-lines" method). These two possibilities are schematized in Figure 15.

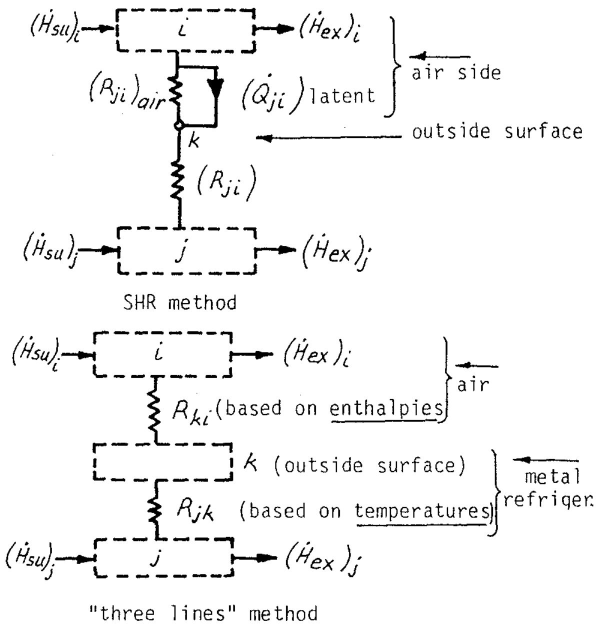

Figure 15 : Cooling coil models

HEAT EXCHANGERS WITH COMBUSTION (boilers, furnaces, ...)

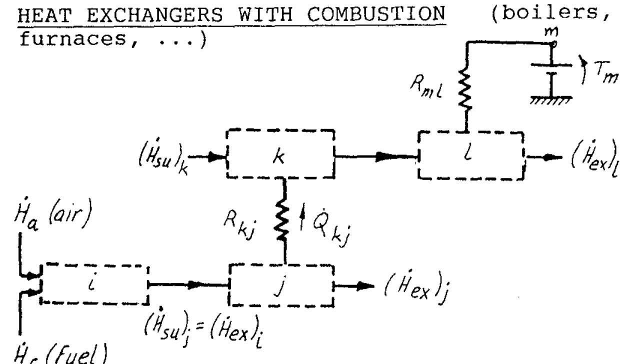

Figure 16 : Static boiler model

This model includes 4 reference volumes :

1) an adiabatic combustion chamber (i) :

$(\dot{H}_{ex})_i = (\dot{H}_{su})_i = \dot{H}_a + \dot{H}_f$
and
$(T_{ex})_i = (\dot{H}_{ex})_i/(\dot{C}_{ex})_i$;

2) a gas chanel (j) :

$(\dot{H}_{ex})_j = (\dot{H}_{su})_j - \dot{Q}_{kj}$;

3) a water chanel (k) :

$(\dot{H}_{ex})_k = (\dot{H}_{su})_k + \dot{Q}_{kj}$

j and k are interconnected through the heat transfer resistance R_{kj}. They constitute the main heat exchanger;

4) a (fictious) secondary water chanel (l) connected to the boiler surroundings (m) :

$(\dot{H}_{ex})_l = (\dot{H}_{su})_l - \dot{Q}_{ml}$

This very simple model allows a characterization of the boiler by two parameters, R_{kj} and R_{ml}, which generally vary strongly according to the burner power rate.

Even more simplification may be introduced when the control mode is clearly identified.

If, for example, the water exhaust temperature $(T_{ex})_l$ is kept constant through an aquastat, the sketch may be simplified as indicated in Figure 17.

We get :
$\dot{Q}_l : (\dot{H}_{ex})_l - (\dot{H}_{su})_l$

$\dot{Q}_l = \dot{C}_l \ (T_{ex} - T_{su})_l$ (19)
and
$\eta = \dot{Q}_l \ / \ \dot{M}_f(PC)_f = f \ (\dot{Q}_l)$ (20)

with : $\dot{M}_f$ = fuel flow rate
PC = calorific value
f = function easily determined by submitting the model of Figure 16 to the control mode considered (ON/OFF cycles or burner power modulation).

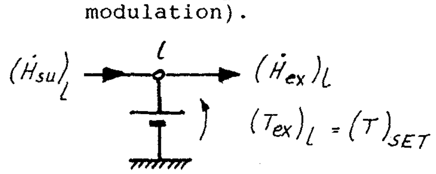

Figure 17 : Constant temperature mode

FLUID "PROPELLERS" (compressors, fans, pumps)

They may be considered as pressure or mass flow sources according to their characteristics and domain of use (Figure 18).

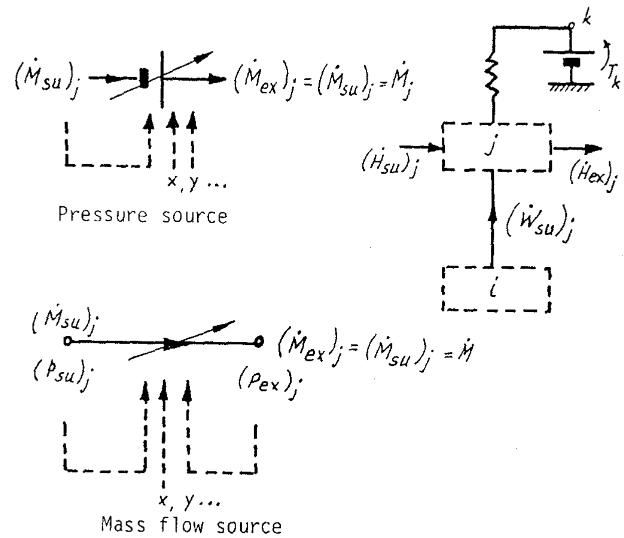

Figure 18 : Models of fluid propellers

The main characteristics :

$p_{ex} = f \ (p_{su}, \dot{M}, \ldots)$
or
$\dot{M} = f \ (p_{su}, p_{ex}, \ldots)$
and
$\dot{W}_{su} = f \ (\dot{M}, \ldots)$

are currently defined through adimensional relationship :

- through flow rate, enthalpy and power factors (φ, ψ, s);
- or through "volumetric" and "isentropic" effectiveness ...

Linearized relationships between mass flow and pressure differences may sometimes be used in limited domains of applications (Figure 19).

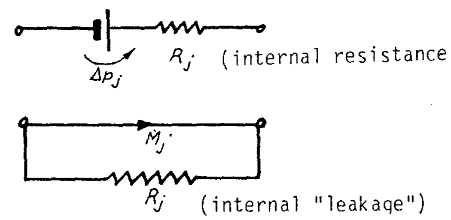

Figure 19 : Linearized models

BUILDINGS AND HVAC SYSTEMS

They may be described as combinations of the components already presented.

AKNOWLEDGEMENT

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